RTI Toolkit: A Practical Guide for Schools

RTI: Resource Materials for Math Interventions & Assessment

Jim Wright, Presenter

9 September 2008
SSTAGE Fall Conference
Dublin, GA

Jim Wright
364 Long Road
Tully, NY 13159
Email: jim@jimwrightonline.com
Website: www.interventioncentral.org
Curriculum-Based Measurement Administration & Scoring
Guidelines for Math Computation

CBM MATH
Description
There are 2 types of CBM math probes, single-skill worksheets (those containing like problems) and multiple-skill worksheets (those containing a mix of problems requiring different math operations). Single-skill probes give instructors good information about students' mastery of particular problem-types, while multiple-skill probes allow the teacher to test children's math competencies on a range of computational objectives during a single CBM session.

Figure 5: A Sampling of Math Computational Goals for Addition, Subtraction, Multiplication, and Division (from Wright, 2002).

Addition
Two 1-digit numbers: sums to 10
Two 3-digit numbers: no regrouping
1- to 2-digit number plus 1- to 2-digit number: regrouping

Subtraction
Two 1-digit numbers: 0 to 9
2-digit number from a 2-digit number: no regrouping
2-digit number from a 2-digit number: regrouping

Multiplication
Multiplication facts: 0 to 9
2-digit number times 1-digit number: no regrouping
3-digit number times 1-digit number: regrouping

Division
Division facts: 0 to 9
2-digit number divided by 1-digit number: no remainder
2-digit number divided by 1-digit number: remainder


Both types of math probes can be administered either individually or to groups of students. The examiner hands the worksheet(s) out to those students selected for assessment. Next, the examiner reads aloud the directions for the worksheet. Then the signal is given to start, and students proceed to complete as many items as possible within 2 minutes. The examiner collects the worksheets at the end of the assessment for scoring.

Creating a measurement pool for math computational probes
The first task of the instructor in preparing CBM math probes is to define the computational skills to be assessed. Many districts have adopted their own math curriculum that outlines the various computational
skills in the order in which they are to be taught. Teachers may also review scope-and-sequence charts that accompany math textbooks when selecting CBM computational objectives.

The order in which math computational skills are taught, however, probably does not vary a great deal from district to district. Figure 5 contains sample computation goals for addition, subtraction, multiplication, and division.

Instructors typically are interested in employing CBM to monitor students' acquisition of skills in which they are presently being instructed. However, teachers may also want to use CBM as a skills check-up to assess those math objectives that students have been taught in the past or to "preview" a math group's competencies in computational material that will soon be taught.

**Preparing CBM Math Probes**

After computational objectives have been selected, the instructor is ready to prepare math probes. The teacher may want to create single-skills probes, multipleskill probes, or both types of CBM math worksheets.

**Creating the Single-skill Math Probe**

As the first step in putting together a single-skill math probe, the teacher will select one computational objective as a guide. The measurement pool, then, will consist of problems randomly constructed that conform to the computational objective chosen. For example, the instructor may select the following computational objective (Figure 6) as the basis for a math probe.

The teacher would then construct a series of problems that match the computational goal, as in Figure 6. In general, single-skill math probes should contain between 80 and 200 problems, and worksheets should have items on both the front and back of the page. Adequate space should also be left for the student's computations, especially with more complex problems such as long division.

**Creating the Multiple-skill Math Probe**

To assemble a multiple-skill math probe, the instructor will first select the range of math operations and of problem-types that will make up the probe. The teacher will probably want to consult the district math

---

**Figure 6: Example of a single-skill math probe: Three to five 3- and 4-digit numbers: no regrouping**

<table>
<thead>
<tr>
<th>105</th>
<th>2031</th>
<th>111</th>
<th>634</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 600</td>
<td>+ 531</td>
<td>+ 717</td>
<td>+ 8240</td>
</tr>
<tr>
<td>+ 293</td>
<td>+ 2322</td>
<td>+ 260</td>
<td>+ 203</td>
</tr>
</tbody>
</table>

---

**Figure 7: Example of a multiple-skill math probe:**

Division: 3-digit number divided by 1-digit number: no remainder
Subtraction: 2-digit number from a 2-digit number: regrouping
Multiplication: 3-digit number times 1-digit number: no regrouping
Division: Two 3-digit numbers: no regrouping

<table>
<thead>
<tr>
<th>9/431</th>
<th>20</th>
<th>113</th>
<th>106</th>
</tr>
</thead>
<tbody>
<tr>
<td>-18</td>
<td>x 2</td>
<td></td>
<td>+ 172</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+ 200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+ 600</td>
</tr>
</tbody>
</table>
curriculum, appropriate scope—and sequence charts, or the computational-goal chart included in this manual when selecting the kinds of problems to include in the multiple-skill probe. Once the computational objectives have been chosen, the teacher can make up a worksheet of mixed math facts conforming to those objectives. Using our earlier example, the teacher who wishes to estimate the proficiency of his 4th-grade math group may decide to create a multiple-skills CBM probe. He could choose to sample only those problem-types that his students have either mastered or are presently being instructed in. Those skills are listed in Figure 7, with sample problems that might appear on the worksheet of mixed math facts.

Materials needed for giving CBM math probes

Student copy of CBM math probe (either single- or multiple-skill)

Stopwatch

Pencils for students

Administration of CBM math probes

The examiner distributes copies of one or more math probes to all the students in the group. (Note: These probes may also be administered individually). The examiner says to the students:

The sheets on your desk are math facts.

If the students are to complete a single-skill probe, the examiner then says: All the problems are [addition or subtraction or multiplication or division] facts.

If the students are to complete a multiple-skill probe, the examiner then says: There are several types of problems on the sheet. Some are addition, some are subtraction, some are multiplication, and some are division [as appropriate]. Look at each problem carefully before you answer it.

When I say 'start,' turn them over and begin answering the problems. Start on the first problem on the left on the top row [point]. Work across and then go to the next row. If you can't answer the problem, make an 'X' on it and go to the next one. If you finish one side, go to the back. Are there any questions? Say, Start.

The examiner starts the stopwatch. While the students are completing worksheets, the examiner and any other adults assisting in the assessment circulate around the room to ensure that students are working on the correct sheet, that they are completing problems in the correct order (rather than picking out only the easy items), and that they have pencils, etc.

After 2 minutes have passed, the examiner says Stop. CBM math probes are collected for scoring.

Scoring

Traditional approaches to computational assessment usually give credit for the total number of correct answers appearing on a worksheet. If the answer to a problem is found to contain one or more incorrect digits, that problem is marked wrong and receives no credit. In contrast to this all-or-nothing marking system, CBM assigns credit to each individual correct digit appearing in the solution to a math fact.

On the face of it, a math scoring system that awards points according to the number of correct digits may appear unusual, but this alternative approach is grounded in good academic-assessment research and practice. By separately scoring each digit in the answer of a computation problem, the instructor is better able to recognize and to give credit for a student's partial math competencies. Scoring computation problems by the digit rather than as a single answer also allows for a more minute analysis of a child's number skills.

Imagine, for instance, that a student was given a CBM math probe consisting of addition problems, sums less than or equal to 19 (incorrect digits appear in boldface and italics):

Figure 8: Example of completed problems from a single-skill math probe

<table>
<thead>
<tr>
<th>105</th>
<th>2031</th>
<th>111</th>
<th>634</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 600</td>
<td>+ 531</td>
<td>+ 717</td>
<td>+ 8240</td>
</tr>
<tr>
<td>+ 293</td>
<td>+ 2322</td>
<td>+ 260</td>
<td>+ 203</td>
</tr>
<tr>
<td>988</td>
<td>4884</td>
<td>1087</td>
<td>9077</td>
</tr>
</tbody>
</table>
If the answers in Figure 8 were scored as either correct or wrong, the child would receive a score of 1 correct answer out of 4 possible answers (25 percent). However, when each individual digit is scored, it becomes clear that the student actually correctly computed 12 of 15 possible digits (80 percent). Thus, the CBM procedure of assigning credit to each correct digit demonstrates itself to be quite sensitive to a student’s emerging, partial competencies in math computation.

The following scoring rules will aid the instructor in marking single- and multiple-skill math probes:

- Individual correct digits are counted as correct. Reversed or rotated digits are not counted as errors unless their change in position makes them appear to be another digit (e.g., 9 and 6).

- Incorrect digits are counted as errors. Digits that appear in the wrong place value, even if otherwise correct, are scored as errors.

  Example
  
  \[
  \begin{array}{c}
  97 \\
  \times 9 \\
  \hline
  8730
  \end{array}
  \]

  “873” is the correct answer to this problem, but no credit can be given since the addition of the 0 pushes the other digits out of their proper place-value positions.

- The student is given credit for “place-holder” numerals that are included simply to correctly align the problem. As long as the student includes the correct space, credit is given whether or not a “0” has actually been inserted.

  Example
  
  \[
  \begin{array}{c}
  55 \\
  \times 82 \\
  \hline
  110 \\
  4400 \\
  4510
  \end{array}
  \]

  Since the student correctly placed 0 in the “place-holder” position, it is given credit as a correct digit. Credit would also have been given if the space were reserved but no 0 had been inserted.

- In more complex problems such as advanced multiplication, the student is given credit for all correct numbers that appear below the line.

  Example
  
  \[
  \begin{array}{c}
  33 \\
  \times 28 \\
  \hline
  264 \\
  660 \\
  924
  \end{array}
  \]

  Credit is given for all work below the line. In this example, the student earns credit for 9 correct digits.

- Credit is not given for any numbers appearing above the line (e.g., numbers marked at the top of number columns to signify regrouping).

  Example
  
  \[
  \begin{array}{c}
  1 \\
  46 \\
  + 39 \\
  \hline
  85
  \end{array}
  \]

  Credit is given for the 2 digits below the line. However, the carried “1” above the line does not receive credit.

APPENDIX D: List of computational goals

COMPUTATIONAL GOALS OF MATH CURRICULUM (ADAPTED FROM SHAPIRO, 1989)

The computational skills listed below are arranged in ascending order of difficulty. Please identify (1) the skills which you have instructed in the classroom, (2) the skills that the student has mastered, and (3) the skills with which the student is currently having difficulty.

**MASTERED** : Place a check under the M column indicating the skills which the student has mastered.

**INSTRUCTED** : Place a check under the I column indicating the skills which you have instructed.

**DIFFICULTY** : Place a check under the D column indicating the skills with which the student is having difficulty.

<table>
<thead>
<tr>
<th>M</th>
<th>I</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Grade 1**

1. Add two one-digit numbers: sums to 10.
2. Subtract two one-digit numbers: combinations to 10.

**Grade 2**

3. Add two one-digit numbers: sums 11 to 19.
4. Add a one-digit number to a two-digit number--no regrouping.
5. Add a two-digit number to a two-digit number--no regrouping.
6. Add a three-digit number to a three-digit number--no regrouping.
7. Subtract a one-digit number from a one- or two-digit number: combinations to 18.
8. Subtract a one-digit number from a two-digit number--no regrouping.
9. Subtract a two-digit number from a two-digit number--no regrouping.
10. Subtract a three-digit number from a three-digit number--no regrouping.
11. Multiplication facts--0's, 1's, 2's.

**Grade 3**

12. Add three or more one-digit numbers.
13. Add three or more two-digit numbers--no regrouping.
14. Add three or more three- and four-digit numbers--no regrouping.
15. Add a one-digit number to a two-digit number with regrouping.
16. Add a two-digit number to a two-digit number with regrouping.
17. Add a two-digit number to a three-digit number with regrouping from the units to the tens column only.
18. Add a two-digit number to a three-digit number with regrouping from the tens to the hundreds column only.
19. Add a two-digit number to a three-digit number with regrouping from the units to the tens column and from the tens to the hundreds column.
20. Add a three-digit number to a three-digit number with regrouping from the units to the tens column only.
21. Add a three-digit number to a three-digit number with regrouping from the tens to the hundreds column only.
Appendix D: Computational Goals

M   I   D

__ __ __ 22. Add a three-digit number to a three-digit number with regrouping from the units to the tens column and from the tens to the hundreds column.
__ __ __ 23. Add a four-digit number to a four-digit number with regrouping in one to three columns.
__ __ __ 24. Subtract two four-digit numbers-no regrouping.
__ __ __ 25. Subtract a one-digit number from a two-digit number with regrouping.
__ __ __ 26. Subtract a two-digit number from a two-digit number with regrouping.
__ __ __ 27. Subtract a two-digit number from a three-digit number with regrouping from the units to the tens column only.
__ __ __ 28. Subtract a two-digit number from a three-digit number with regrouping from the tens to the hundreds column only.
__ __ __ 29. Subtract a two-digit number from a three-digit number with regrouping from the units to the tens column and from the tens to the hundreds column.
__ __ __ 30. Subtract a three-digit from a three-digit number with regrouping from the units to the tens column only.
__ __ __ 31. Subtract a three-digit number from a three-digit number with regrouping from the tens to the hundreds column only.
__ __ __ 32. Subtract a three-digit number from a three-digit number with regrouping from the units to the tens column and from the tens to the hundreds column.
__ __ __ 33. Multiplication facts--3 to 9.

Grade 4

__ __ __ 34. Add a five- or six-digit number to a five- or six-digit number with regrouping in any columns.
__ __ __ 35. Add three or more two-digit numbers with regrouping.
__ __ __ 36. Add three or more three-digit numbers with regrouping with regrouping in any columns.
__ __ __ 37. Subtract a five- or six-digit number from a five- or six-digit number with regrouping in any columns.
__ __ __ 38. Multiply a two-digit number by a one-digit number with no regrouping.
__ __ __ 39. Multiply a three-digit number by a one-digit number with no regrouping.
__ __ __ 40. Multiply a two-digit number by a one-digit number with no regrouping.
__ __ __ 41. Multiply a three-digit number by a one-digit number with regrouping.
__ __ __ 42. Division facts--0 to 9.
__ __ __ 43. Divide a two-digit number by a one-digit number with no remainder.
__ __ __ 44. Divide a two-digit number by a one-digit number with remainder.
__ __ __ 45. Divide a three-digit number by a one digit number with remainder.
__ __ __ 46. Divide a four-digit number by a one-digit number with remainder.
## Appendix D: Computational Goals

<table>
<thead>
<tr>
<th>M</th>
<th>I</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

47. Multiply a two-digit number by a two-digit number with regrouping.

48. Multiply a three-digit number by a two-digit number with regrouping.

49. Multiply a three-digit number by a three-digit number with regrouping.

Curriculum-Based Assessment Mathematics
Multiple-Skills Computation Probe: Student Copy

Student: __________________________

Date: __________________________

727,162 + 30,484 = 42,286 - 29,756 = 156 x 623 = 52,2207

146,569 + 532,260 = 33,516 - 21,366 = 192 x 371 = 43,4742

www.interventioncentral.org
Curriculum-Based Assessment Mathematics  
Multiple-Skills Computation Probe: Examiner Copy

Item 1:  
6 CD/6 CD Total  
ADDITION: 5- to 6-digit number plus 5- to 6-digit number:  
Regrouping in any column  

727,162  
+ 30,484  
______  
757,646

Item 2:  
5 CD/11 CD Total  
SUBTRACTION: 5-digit number from 5-digit number:  
Regrouping in any column  

42,286  
- 29,756  
______  
12,530

Item 3:  
17 CD/28 CD Total  
MULTIPLICATION: 3-digit number times 3-digit number:  
Regrouping  

156  
x 623  
______  
468  
312  
936--  
97,188

Item 4:  
15 CD/43 CD Total  
DIVISION: 4-digit number divided by 2-digit number: remainder  

42  
43  
52  
-208  
127  
-104  
23

Item 5:  
6 CD/49 CD Total  
ADDITION: 5- to 6-digit number plus 5- to 6-digit number:  
Regrouping in any column  

146,569  
+532,260  
______  
678,829

Item 6:  
5 CD/54 CD Total  
SUBTRACTION: 5-digit number from 5-digit number:  
Regrouping in any column  

33,516  
- 21,366  
______  
12,150

Item 7:  
18 CD/72 CD Total  
MULTIPLICATION: 3-digit number times 3-digit number:  
Regrouping  

192  
x 371  
______  
192  
1344--  
71,232

Item 8:  
13 CD/85 CD Total  
DIVISION: 4-digit number divided by 2-digit number: remainder  

110  
43  
4742  
-43  
44  
-43  
12

www.interventioncentral.org
Setting up the graph

• At the top of the graph, fill out the student's name, his or her classroom and/or grade, and information about the level at which the student is being monitored with CBM.

• After you have collected baseline CBM information, fill out the start date and end date in the Baseline date section for the time span during which you collected baseline data (Figure 1). Then decide how many instructional weeks that you plan to monitor the student’s progress. Fill out the start date (Monday) and end date (Friday) in the Monitoring date section for each instructional week during which monitoring will take place (Figure 1). If possible, you should try to collect at least one CBM observation per week for your target student. It is a good idea to fill in the weekly start- and end-dates in advance to give yourself an incentive to stay up-to-date on your CBM monitoring.

Entering information onto the graph

• **Baseline datapoints.** Collect at least 3-5 baseline datapoints. (Baseline data are collected to get a sense of the student's current performance level and rate of progress. It is a good idea to collect them within a 1- to 2-week span.) Plot these datapoints in the ‘baseline’ column on the graph, as shown in Figure 2. Next to each plotted datapoint, write the date on which it was collected. Connect all baseline datapoints with lines to identify them as a single data-series.

• **Progress-monitoring datapoints.** When graphing a CBM datapoint collected during progress monitoring, find the week whose date span includes the date on which the CBM assessment was completed. At the bottom of the graph, circle the weekday ('MTWTF') on which the assessment was conducted. Then plot the datapoint above that circled day. (See Figure 3 for an example.) Connect all monitoring datapoints with lines to identify them as a single data-series. Do not connect the baseline and monitoring data-series, however, as each should be considered separate data ‘phases’.

Want additional guidelines for setting up your data chart?
| Student: _____________________ | Classrm/Grade: _____________________ | Monitoring Level: ____________ |

| Instructional Days | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Baseline | | | | | | | | | | | | | | |
| MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF | MTWF |

Correct Digits Per 2 Minutes: Problem Type(s):

Instructional 1-3
Instructional 4+
Mastery 1-3
Mastery 4+
Frustrational 1-3
Frustrational 4+

Baseline
Week 1
Week 2
Week 3
Week 4
Week 5
Week 6
Week 7
Week 8
Week 9
Week 10
Week 11
Week 12

Math 80-12 ©2003 Jim Wright www.interventioncentral.org
Early Math Fluency CBM Probe: Quantity Discrimination

This introduction to the Quantity Discrimination probe provides information about the preparation, administration, and scoring of this Early Math CBM measure. Additionally, it offers brief guidelines for integrating this assessment into a school-wide ‘Response-to-Intervention’ model.

**Quantity Discrimination: Description** (Clarke & Shinn, 2005; Gersten, Jordan & Flojo, 2005)
The student is given a sheet containing pairs of numbers. In each number pair, one number is larger than the other. The numbers in each pair are selected from within a predefined range (e.g., no lower than 0 and no higher than 20). During a one-minute timed assessment, the student identifies the larger number in each pair, completing as many items as possible while the examiner records any Quantity Discrimination errors.

**Quantity Discrimination: Preparation**
The following materials are needed to administer Quantity Discrimination (QD) Early Math CBM probes:

- Student and examiner copies of a QD assessment probe. *(Note: Customized QD probes can be created conveniently and at no cost using Numberfly, a web-based application. Visit Numberfly at http://www.interventioncentral.org/php/numberfly/numberfly.php).*
- A pencil, pen, or marker
- A stopwatch

**Quantity Discrimination: Directions for Administration**

1. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.

2. The examiner says to the student:

   “The sheet on your desk has pairs of numbers. In each set, one number is bigger than the other.”

   “When I say, 'start,' tell me the name of the number that is larger in each pair. Start at the top of this page and work across the page [demonstrate by pointing]. Try to figure out the larger number for each example. When you come to the end of a row, go to the next row. Are there any questions? [Pause] Start.”

3. The examiner begins the stopwatch when the student responds aloud to the first item. If the student hesitates on a number for 3 seconds or longer on a Quantity Discrimination item, the examiner says, “Go to the next one.” (If necessary, the examiner points to the next number as a student prompt.)
4. The examiner marks each Quantity Discrimination error by marking a slash (/) through the incorrect response item on the examiner form.

5. At the end of one minute, the examiner says, “Stop” and writes in a right-bracket symbol ( ] ) on the examiner form after the last item that the student had attempted when the time expired. The examiner then collects the student Quantity Discrimination sheet.

**Quantity Discrimination: Scoring Guidelines**

Correct QD responses include:

- Quantity Discriminations read correctly
- Quantity Discriminations read incorrectly but corrected by the student within 3 seconds

Incorrect QD responses include:

- The student’s reading the smaller number in the QD number pair
- Correct QD responses given after hesitations of 3 seconds or longer
- The student’s calling out a number other than appears in the QD number pair
- Response items skipped by the student

To calculate a Quantity Discrimination fluency score, the examiner:

1. counts up all QD items that the student attempted to answer and
2. subtracts the number of QD errors from the total number attempted.
3. The resulting figure is the number of correct Quantity Discrimination items completed (QD fluency score).

**Quantity Discrimination Probes as Part of a Response to Intervention Model**

- **Universal Screening:** To proactively identify children who may have deficiencies in development of foundation math concepts, or ‘number sense’ (Berch, 2003), schools may choose to screen all kindergarten and first grade students using Quantity Discrimination probes. Those screenings would take place in fall, winter, and spring. Students who fall below the ‘cutpoint’ of the 35th percentile (e.g., Jordan & Hanich, 2003) of the grade norms on the QD task would be identified as having moderate deficiencies and given additional interventions to build their ‘number sense’ skills.

- **Tier I (Classroom-Based) Interventions:** Teachers can create Quantity Discrimination probes and use them independently to track the progress of students who show modest delays in their math foundation skills.

- **Tier II (Individualized) Interventions:** Students with more extreme academic delays may be referred to a school-based problem-solving team, which will develop more intensive, specialized interventions to target the student’s academic deficits (Wright, 2007). Quantity Discrimination probes can be used as one formative measure to track student progress with Tier II interventions to build foundation math skills.
Quantity Discrimination: Measurement Statistics

Test-Retest Reliability Correlations for Quantity Discrimination Probes

<table>
<thead>
<tr>
<th>Time Span</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-week interval</td>
<td>0.85</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
<tr>
<td>26-week interval</td>
<td>0.86</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
</tbody>
</table>

Predictive Validity Correlations for Quantity Discrimination Probes

<table>
<thead>
<tr>
<th>Predictive Validity Measure</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum-Based Measurement Math Computation Fluency Probes: Grade 1 Addition &amp; Subtraction (Fall Administration of QD Probe and Spring Administration of Math Computation Probe)</td>
<td>0.67</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
<tr>
<td>Woodcock-Johnson Tests of Achievement: Applied Problems subtest (Fall Administration of QD Probe and Spring Administration of WJ-ACH subtest)</td>
<td>0.79</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
</tbody>
</table>

References


Correct Quantity Discrimination Items Identified Per Min from Range of ____ to ____

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASELINE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEEK 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Instructional Days

©2007 Jim Wright www.interventioncentral.org
Early Math Fluency CBM Probe: Missing Number

This introduction to the Missing Number probe provides information about the preparation, administration, and scoring of this Early Math CBM measure. Additionally, it offers brief guidelines for integrating this assessment into a school-wide ‘Response-to-Intervention’ model.

**Missing Number: Description** (Clarke & Shinn, 2005; Gersten, Jordan & Flojo, 2005)
The student is given a sheet containing multiple number series. Each series consists of 3-4 numbers that appear in sequential order. The numbers in each short series are selected to fall within a predefined range (e.g., no lower than 0 and no higher than 20). In each series, one number is left blank (e.g., ‘1 2 _ 4’). During a one-minute timed assessment, the student states aloud the missing number in as many response items as possible while the examiner records any Missing Number errors.

**Missing Number: Preparation**
The following materials are needed to administer Missing Number (MN) Early Math CBM probes:

- Student and examiner copies of a MN assessment probe. *(Note: Customized MN probes can be created conveniently and at no cost using Numberfly, a web-based application. Visit Numberfly at [http://www.interventioncentral.org/php/numberfly/numberfly.php](http://www.interventioncentral.org/php/numberfly/numberfly.php)).*

- A pencil, pen, or marker

- A stopwatch

**Missing Number: Directions for Administration**
1. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.

2. The examiner says to the student:

   “The sheet on your desk has sets of numbers. In each set, a number is missing.”

   “When I say, ‘start,’ tell me the name of the number that is missing from each set of numbers. Start at the top of this page and work across the page [demonstrate by pointing]. Try to figure out the missing number for each example. When you come to the end of a row, go to the next row. Are there any questions? [Pause] Start “

3. The examiner begins the stopwatch when the student reads the first number aloud. If the student hesitates on a number for 3 seconds or longer on a Missing Number item, the examiner says the correct number aloud and says, “Go to the next one.” (If necessary, the examiner points to the next number as a student prompt.)

4. The examiner marks each Missing Number error by marking a slash (/) through the incorrect response item on the examiner form.
5. At the end of one minute, the examiner says, “Stop” and writes in a right-bracket symbol ( ] ) on the examiner form after the last item that the student had attempted when the time expired. The examiner then collects the student Missing Number sheet.

**Missing Number: Scoring Guidelines**
Correct MN responses include:

- Missing numbers read correctly
- Missing numbers read incorrectly but corrected by the student within 3 seconds

Incorrect MN responses include:

- Missing numbers read incorrectly
- Missing numbers read correctly after hesitations of 3 seconds or longer
- Response items skipped by the student

To calculate a Missing Number fluency score, the examiner:

1. counts up all MN items that the student attempted to read aloud and
2. subtracts the number of MN errors from the total number attempted.
3. The resulting figure is the number of correct Missing Number items completed. (MN fluency score).

**Missing Number Probes as Part of a Response to Intervention Model**

- Universal Screening: To proactively identify children who may have deficiencies in development of foundation math concepts, or ‘number sense’ (Berch, 2003), schools may choose to screen all kindergarten and first grade students using Missing Number probes. Those screenings would take place in fall, winter, and spring. Students who fall below the ‘cutpoint’ of the 35th percentile (e.g., Jordan & Hanich, 2003) of the grade norms on the MN task would be identified as having moderate deficiencies and given additional interventions to build their ‘number sense’ skills.

- Tier I (Classroom-Based) Interventions: Teachers can create Missing Number probes and use them independently to track the progress of students who show modest delays in their math foundation skills.

- Tier II (Individualized) Interventions. Students with more extreme academic delays may be referred to a school-based problem-solving team, which will develop more intensive, specialized interventions to target the student’s academic deficits (Wright, 2007). Missing Number probes can be used as one formative measure to track student progress with Tier II interventions to build foundation math skills.

**Missing Number: Measurement Statistics**

<table>
<thead>
<tr>
<th>Time Span</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-week interval</td>
<td>0.79</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
</tbody>
</table>
26-week interval | 0.81 | Clarke & Shinn (2005)

<table>
<thead>
<tr>
<th>Predictive Validity Measure</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum-Based Measurement Math Computation Fluency Probes: Grade 1 Addition &amp; Subtraction (Fall Administration of MN Probe and Spring Administration of Math Computation Probe)</td>
<td>0.67</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
<tr>
<td>Woodcock-Johnson Tests of Achievement: Applied Problems subtest (Fall Administration of MNF Probe and Spring Administration of WJ-ACH subtest)</td>
<td>0.72</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
</tbody>
</table>

References


Correct Missing Numbers Identified Per Minute from Range of __ to __

<table>
<thead>
<tr>
<th>Instructional Days</th>
<th>Baseline</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
<th>Week 9</th>
<th>Week 10</th>
<th>Week 11</th>
<th>Week 12</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Early Math Fluency CBM Probe: Number Identification

This introduction to the Number Identification probe provides information about the preparation, administration, and scoring of this Early Math CBM measure. Additionally, it offers brief guidelines for integrating this assessment into a school-wide ‘Response-to-Intervention’ model.

**Number Identification: Description** (Clarke & Shinn, 2005; Gersten, Jordan & Flojo, 2005)
The student is given a sheet containing rows of randomly generated numbers (e.g., ranging from 0 to 20). During a one-minute timed assessment, the student reads aloud as many numbers as possible while the examiner records any Number Identification errors.

**Number Identification: Preparation**
The following materials are needed to administer Number Identification (NID) Early Math CBM probes:

- Student and examiner copies of a NID assessment probe. *(Note: Customized NID probes can be created conveniently and at no cost using Numberfly, a web-based application. Visit Numberfly at [http://www.interventioncentral.org/php/numberfly/numberfly.php](http://www.interventioncentral.org/php/numberfly/numberfly.php)).*
- A pencil, pen, or marker
- A stopwatch

**Number Identification: Directions for Administration**
1. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.

2. The examiner says to the student:

   "The sheet on your desk has rows of numbers."

   "When I say, 'start,' begin reading the numbers aloud. Start at the top of this page and read across the page [demonstrate by pointing]. Try to read each number. When you come to the end of a row, go to the next row. Are there any questions? [Pause] Start."

3. The examiner begins the stopwatch when the student reads the first number aloud. If the student hesitates on a number for 3 seconds or longer, the examiner says, "Go to the next one." (If necessary, the examiner points to the next number as a student prompt.)

4. The examiner marks each Number Identification error by marking a slash (/) through the incorrectly read number on the examiner form.

5. At the end of one minute, the examiner says, “Stop” and writes in a right-bracket symbol ( [ ] ) on the examiner form from the point in the number series that the student had reached when the time expired. The examiner then collects the student Number Identification sheet.
Number Identification: Scoring Guidelines
Correct NID responses include:

- Numbers read correctly
- Numbers read incorrectly but corrected by the student within 3 seconds

Incorrect NID responses include:

- Numbers read incorrectly
- Numbers read correctly after hesitations of 3 seconds or longer
- Numbers skipped by the student

To calculate a Number Identification fluency score, the examiner:

1. counts up all numbers that the student attempted to read aloud and
2. subtracts the number of errors from the total of numbers attempted.
3. The resulting figure is the number of correct numbers identified. (NID fluency score).

Number Identification Probes as Part of a Response to Intervention Model

- Universal Screening: To proactively identify children who may have deficiencies in development of foundation math concepts, or ‘number sense’ (Berch, 2003), schools may choose to screen all kindergarten and first grade students using Number Identification probes. Those screenings would take place in fall, winter, and spring. Students who fall below the ‘cutpoint’ of the 35th percentile (e.g., Jordan & Hanich, 2003) of the grade norms on the NID task would be identified as having moderate deficiencies and given additional interventions to build their ‘number sense’ skills.

- Tier I (Classroom-Based) Interventions: Teachers can create Number Identification probes and use them independently to track the progress of students who show modest delays in their math foundation skills.

- Tier II (Individualized) Interventions. Students with more extreme academic delays may be referred to a school-based problem-solving team, which will develop more intensive, specialized interventions to target the student’s academic deficits (Wright, 2007). Number Identification probes can be used as one formative measure to track student progress with Tier II interventions to build foundation math skills.

Number identification: Measurement Statistics

<table>
<thead>
<tr>
<th>Test-Retest Reliability Correlations for Number Identification Probes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Span</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>13-week interval</td>
</tr>
<tr>
<td>26-week interval</td>
</tr>
</tbody>
</table>
### Predictive Validity Correlations for Number Identification Probes

<table>
<thead>
<tr>
<th>Predictive Validity Measure</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum-Based Measurement Math Computation Fluency Probes: Grade 1 Addition &amp; Subtraction (Fall Administration of MN Probe and Spring Administration of Math Computation Probe)</td>
<td>0.60</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
<tr>
<td>Woodcock-Johnson Tests of Achievement: Applied Problems subtest (Fall Administration of NID Probe and Spring Administration of WJ-ACH subtest)</td>
<td>0.72</td>
<td>Clarke &amp; Shinn (2005)</td>
</tr>
</tbody>
</table>

### References


<table>
<thead>
<tr>
<th></th>
<th>BASELINE</th>
<th>WEEK 1</th>
<th>WEEK 2</th>
<th>WEEK 3</th>
<th>WEEK 4</th>
<th>WEEK 5</th>
<th>WEEK 6</th>
<th>WEEK 7</th>
<th>WEEK 8</th>
<th>WEEK 9</th>
<th>WEEK 10</th>
<th>WEEK 11</th>
<th>WEEK 12</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>20</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>30</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>40</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>50</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>60</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>70</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>80</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correct Numbers Identified Per Minute from Range of ___ to ___

Instructional Days

WEEK 1: ___-___

WEEK 2: ___-___

WEEK 3: ___-___

WEEK 4: ___-___

WEEK 5: ___-___

WEEK 6: ___-___

WEEK 7: ___-___

WEEK 8: ___-___

WEEK 9: ___-___

WEEK 10: ___-___

WEEK 11: ___-___

WEEK 12: ___-___
Mathematics instruction is a lengthy, incremental process that spans all grade levels. As children begin formal schooling in kindergarten, they develop ‘number sense’, an intuitive understanding of foundation number concepts and relationships among numbers. A central part of number sense is the student’s ability to internalize the number line as a precursor to performing mental arithmetic. As students progress through elementary school, they must next master common math operations (addition, subtraction, multiplication, and division) and develop fluency in basic arithmetic combinations (‘math facts’). In later grades, students transition to applied, or ‘word’, problems that relate math operations and concepts to real-world situations. Successful completion of applied problems requires that the student understand specialized math vocabulary, identify the relevant math operations needed to solve the problem while ignoring any unnecessary information also appearing in that written problem, translate the word problem from text format into a numeric equation containing digits and math symbols, and then successfully solve. It is no surprise, then, that there are a number of potential blockers to student success with applied problems, including limited reading decoding and comprehension skills, failure to acquire fluency with arithmetic combinations (math facts), and lack of proficiency with math operations.

Deciding what specific math interventions might be appropriate for any student must therefore be a highly individualized process, one that is highly dependent on the student’s developmental level and current math skills, the requirements of the school district’s math curriculum, and the degree to which the student possesses or lacks the necessary auxiliary skills (e.g., math vocabulary, reading comprehension) for success in math. Here are some wide-ranging classroom (Tier I RTI) ideas for math interventions that extend from the primary through secondary grades.

**Applied Problems: Encourage Students to Draw to Clarify Understanding** (Van Essen & Hamaker, 1990; Van Garderen, 2006). Making a drawing of an applied, or ‘word’, problem is one easy heuristic tool that students can use to help them to find the solution. An additional benefit of the drawing strategy is that it can reveal to the teacher any student misunderstandings about how to set up or solve the word problem. To introduce students to the drawing strategy, the teacher hands out a worksheet containing at least six word problems. The teacher explains to students that making a picture of a word problem sometimes makes that problem clearer and easier to solve. The teacher and students then independently create drawings of each of the problems on the worksheet. Next, the students show their drawings for each problem, explaining each drawing and how it relates to the word problem. The teacher also participates, explaining his or her drawings to the class or group. Then students are directed independently to make drawings as an intermediate problem-solving step when they are faced with challenging word problems. NOTE: This strategy appears to be more effective when used in later, rather than earlier, elementary grades.

**Applied Problems: Improving Performance Through a 4-Step Problem-Solving Approach** (Pólya, 1957; Williams, 2003). Students can consistently perform better on applied math problems if they follow an efficient 4-step plan of understanding the problem, devising a plan, carrying out the plan, and looking back. (1) UNDERSTAND THE PROBLEM. To fully grasp the problem, the student may restate the problem in his or her own words, note key information, and identify missing information. (2) DEVISE A PLAN. In mapping out a strategy to solve the problem, the student may make a table, draw a diagram, or translate the verbal problem into an equation. (3) CARRY OUT THE PLAN. The student implements the steps in the plan, showing work and checking work for each step. (4) LOOK BACK. The student checks the results. If the answer is written as an equation, the student puts the results in words and checks whether the answer addresses the question posed in the original word problem.
Math Computation: Boost Fluency Through Explicit Time-Drills (Rhymer, Skinner, Jackson, McNeill, Smith & Jackson, 2002; Skinner, Pappas & Davis, 2005; Woodward, 2006). Explicit time-drills are a method to boost students’ rate of responding on math-fact worksheets. The teacher hands out the worksheet. Students are told that they will have 3 minutes to work on problems on the sheet. The teacher starts the stop watch and tells the students to start work. At the end of the first minute in the 3-minute span, the teacher ‘calls time’, stops the stopwatch, and tells the students to underline the last number written and to put their pencils in the air. Then students are told to resume work and the teacher restarts the stopwatch. This process is repeated at the end of minutes 2 and 3. At the conclusion of the 3 minutes, the teacher collects the student worksheets. TIPS: Explicit time-drills work best on ‘simple’ math facts requiring few computation steps. They are less effective on more complex math facts. Also, a less intrusive and more flexible version of this intervention is to use time-prompts while students are working independently on math facts to speed their rate of responding. For example, at the end of every minute of seatwork, the teacher can call the time and have students draw a line under the item that they are working on when that minute expires.

Math Computation: Motivate With ‘Errorless Learning’ Worksheets (Caron, 2007). Reluctant students can be motivated to practice math number problems to build computational fluency when given worksheets that include an answer key (number problems with correct answers) displayed at the top of the page. In this version of an ‘errorless learning’ approach, the student is directed to complete math facts as quickly as possible. If the student comes to a number problem that he or she cannot solve, the student is encouraged to locate the problem and its correct answer in the key at the top of the page and write it in. Such speed drills build computational fluency while promoting students’ ability to visualize and to use a mental number line. TIP: Consider turning this activity into a ‘speed drill’. The student is given a kitchen timer and instructed to set the timer for a predetermined span of time (e.g., 2 minutes) for each drill. The student completes as many problems as possible before the timer rings. The student then graphs the number of problems correctly computed each day on a time-series graph, attempting to better his or her previous score.

Math Computation: Two Ideas to Jump-Start Active Academic Responding (Skinner, Pappas & Davis, 2005). Research shows that when teachers use specific techniques to motivate their classes to engage in higher rates of active and accurate academic responding, student learning rates are likely to go up. Here are two ideas to accomplish increased academic responding on math tasks. First, break longer assignments into shorter assignments with performance feedback given after each shorter ‘chunk’ (e.g., break a 20-minute math computation worksheet task into 3 seven-minute assignments). Breaking longer assignments into briefer segments also allows the teacher to praise struggling students more frequently for work completion and effort, providing an additional ‘natural’ reinforcer. Second, allow students to respond to easier practice items orally rather than in written form to speed up the rate of correct responses.

Math Homework: Motivate Students Through Reinforcers, Interesting Assignments, Homework Planners, and Self-Monitoring (Bryan & Sullivan-Burstein, 1998). Improve students’ rate of homework completion and quality by using reinforcers, motivating ‘real-life’ assignments, a homework planner, and student self-monitoring. (1) Reinforcers: Allow students to earn a small reward (e.g., additional free time) when they turn in all homework assignments for the week. (2) ‘Real-life’ Assignments: Make homework meaningful by linking concepts being taught to students’ lives. In a math lesson on estimating area, for example, give students the homework task of calculating the area of their bedroom and estimating the amount of paint needed to cover the walls. (3) Homework Planner: Teach students to use a homework planner to write down assignments, organize any materials (e.g., worksheets) needed for homework, transport completed homework safely back to school, and provide space for parents and teachers to communicate about homework via written school-home notes. (4) Student Self-Monitoring: Direct students to chart their homework completion each week. Have students plot the number of assignments turned in on-time in green, assignments not turned in at all in red, and assignments turned in late in yellow.
Math Instruction: Consolidate Student Learning During Lecture Through the Peer-Guided Pause (Hawkins, & Brady, 1994). During large-group math lectures, teachers can help students to retain more instructional content by incorporating brief Peer Guided Pause sessions into lectures. Students are trained to work in pairs. At one or more appropriate review points in a lecture period, the instructor directs students to pair up to work together for 4 minutes. During each Peer Guided Pause, students are given a worksheet that contains one or more correctly completed word or number problems illustrating the math concept(s) covered in the lecture. The sheet also contains several additional, similar problems that pairs of students work cooperatively to complete, along with an answer key. Student pairs are reminded to (a) monitor their understanding of the lesson concepts; (b) review the correctly math model problem; (c) work cooperatively on the additional problems, and (d) check their answers. The teacher can direct student pairs to write their names on the practice sheets and collect them as a convenient way to monitor student understanding.

Math Instruction: Increase Student Engagement and Improve Group Behaviors With Response Cards (Armendariz & Umbreit, 1999; Lambert, Cartledge, Heward & Lo, 2006). Response cards can increase student active engagement in group math activities while reducing disruptive behavior. In the group-response technique, all students in the classroom are supplied with an erasable tablet (‘response card’), such as a chalk slate or laminated white board with erasable marker. The teacher instructs at a brisk pace. The instructor first poses a question to the class. Students are given sufficient wait time for each to write a response on his or her response card. The teacher then directs students to present their cards. If most or all of the class has the correct answer, the teacher praises the group. If more than one quarter of the students records an incorrect answer on their cards, however, the teacher uses guided questions and demonstration to steer students to the correct answer.

Math Instruction: Maintain a Supportive Atmosphere for Classroom “Math Talk” (Cooke & Adams, 1998). Teachers can promote greater student ‘risk-taking’ in mathematics learning when they cultivate a positive classroom atmosphere for math discussions while preventing peers from putting each other down. The teacher models behavioral expectations for open, interactive discussions, praises students for their class participation and creative attempts at problem-solving, and regularly points out that incorrect answers and misunderstandings should be celebrated—as they often lead to breakthroughs in learning. The teacher uses open-ended comments (e.g., “What led you to that answer?”) as tools to draw out students and encourage them to explore and apply math concepts in group discussion. Students are also encouraged in a supportive manner to evaluate each other’s reasoning. However, the teacher intervenes immediately to prevent negative student comments or ‘put-downs’ about peers. As with any problem classroom behavior, a first offense requires that the student meet privately with the instructor to discuss teacher expectations for positive classroom behavior. If the student continues to put down peers, the teacher imposes appropriate disciplinary consequences.

Math Instruction: Support Students Through a Wrap-Around Instruction Plan (Montague, 1997; Montague, Warger & Morgan, 2000). When teachers instruct students in more complex math cognitive strategies, they must support struggling learners with a ‘wrap-around’ instructional plan. That plan incorporates several elements: (a) Assessment of the student’s problem-solving skills. The instructor first verifies that the student has the necessary academic competencies to learn higher-level math content, including reading and writing skills, knowledge of basic math operations, and grasp of required math vocabulary. (b) Explicit instruction. The teacher presents new math content in structured, highly organized lessons. The instructor also uses teaching tools such as Guided Practice (moving students from known material to new concepts through a thoughtful series of teacher questions) and ‘overlearning’ (teaching and practicing a skill with the class to the point at which students develop automatic recall and control of it). (c) Process modeling. The teacher adopts a ‘think aloud’ approach, or process modeling, to verbally reveal his or her cognitive process to the class while using a cognitive strategy to solve a math problem. In turn, students are encouraged to think aloud when applying the same strategy—first as part of a whole-class or cooperative learning group, then independently. The teacher observes students
during process modeling to verify that they are correctly applying the cognitive strategy. (d) Performance feedback. Students get regular performance feedback about their level of mastery in learning the cognitive strategy. That feedback can take many forms, including curriculum-based measurement, timely corrective feedback, specific praise and encouragement, grades, and brief teacher conferences. (e) Review of mastered skills or material. Once the student has mastered a cognitive strategy, the teacher structures future class lessons or independent work to give the student periodic opportunities to use and maintain the strategy. The teacher also provides occasional brief ‘booster sessions’, reteaching steps of the cognitive strategy to improve student retention.

Math Instruction: Unlock the Thoughts of Reluctant Students Through Class Journaling (Baxter, Woodward & Olson, 2005). Students can effectively clarify their knowledge of math concepts and problem-solving strategies through regular use of class ‘math journals’. Journaling is a valuable channel of communication about math issues for students who are unsure of their skills and reluctant to contribute orally in class. At the start of the year, the teacher introduces the journaling assignment, telling students that they will be asked to write and submit responses at least weekly to teacher-posed questions. At first, the teacher presents ‘safe’ questions that tap into the students’ opinions and attitudes about mathematics (e.g., ‘How important do you think it is nowadays for cashiers in fast-food restaurants to be able to calculate in their head the amount of change to give a customer?’). As students become comfortable with the journaling activity, the teacher starts to pose questions about the students’ own mathematical thinking relating to specific assignments. Students are encouraged to use numerals, mathematical symbols, and diagrams in their journal entries to enhance their explanations. The teacher provides brief written comments on individual student entries, as well as periodic oral feedback and encouragement to the entire class on the general quality and content of class journal responses. Regular math journaling can prod students to move beyond simple ‘rote’ mastery of the steps for completing various math problems toward a deeper grasp of the math concepts that underlie and explain a particular problem-solving approach. Teachers will find that journal entries are a concrete method for monitoring student understanding of more abstract math concepts. To promote the quality of journal entries, the teacher might also assign them an effort grade that will be calculated into quarterly math report card grades.

Math Problem-Solving: Help Students Avoid Errors With the ‘Individualized Self-Correction Checklist’ (Zrniec Uberti, Mastropieri & Scruggs, 2004). Students can improve their accuracy on particular types of word and number problems by using an ‘individualized self-instruction checklist’ that reminds them to pay attention to their own specific error patterns. To create such a checklist, the teacher meets with the student. Together they analyze common error patterns that the student tends to commit on a particular problem type (e.g., ‘On addition problems that require carrying, I don’t always remember to carry the number from the previously added column.’). For each type of error identified, the student and teacher together describe the appropriate step to take to prevent the error from occurring (e.g., ‘When adding each column, make sure to carry numbers when needed.’). These self-check items are compiled into a single checklist. Students are then encouraged to use their individualized self-instruction checklist whenever they work independently on their number or word problems. As older students become proficient in creating and using these individualized error checklists, they can begin to analyze their own math errors and to make their checklists independently whenever they encounter new problem types.

Math Review: Balance Massed & Distributed Practice (Carnine, 1997). Teachers can best promote students acquisition and fluency in a newly taught math skill by transitioning from massed to distributed practice. When students have just acquired a math skill but are not yet fluent in its use, they need lots of opportunities to try out the skill under teacher supervision—a technique sometimes referred to as ‘massed practice’. Once students have developed facility and independence with that new math skill, it is essential that they then be required periodically to use the skill in order to embed and retain it—a strategy also known as ‘distributed practice’. Teachers can program distributed practice of a math skill such as reducing fractions to least common
denominators into instruction either by (a) regularly requiring the student to complete short assignments in which they practice that skill in isolation (e.g., completing drill sheets with fractions to be reduced), or (b) teaching a more advanced algorithm or problem-solving approach that incorporates—and therefore requires repeated use of—the previously learned math skill (e.g., requiring students to reduce fractions to least-common denominators as a necessary first step to adding the fractions together and converting the resulting improper fraction to a mixed number).

**Math Review: Teach Effective Test-Preparation Strategies** (Hong, Sas, & Sas, 2006). A comparison of the methods that high and low-achieving math students typically use to prepare for tests suggests that struggling math students need to be taught (1) specific test-review strategies and (2) time-management and self-advocacy skills. Among review-related strategies, deficient test-takers benefit from explicit instruction in how to take adequate in-class notes; to adopt a systematic method to review material for tests (e.g., looking over their notes each night, rereading relevant portions of the math text, reviewing handouts from the teacher, etc.), and to give themselves additional practice in solving problems (e.g., by attempting all homework items, tackling additional problems from the text book, and solving problems included in teacher handouts). Deficient test-takers also require pointers in how to allocate and manage their study time wisely, to structure their study environment to increase concentration and reduce distractions, as well as to develop ‘self-advocacy’ skills such as seeking additional help from teachers when needed. Teachers can efficiently teach effective test-preparation methods as a several-session whole-group instructional module.

**Math Vocabulary: Preteach, Model, and Use Standard Math Terms** (Chard, D., n.d.). Three strategies can help students to learn essential math vocabulary: preteaching key vocabulary items, modeling those vocabulary words, and using only universally accepted math terms in instruction. (1) Preteach key math vocabulary. Math vocabulary provides students with the language tools to grasp abstract mathematical concepts and to explain their own reasoning. Therefore, do not wait to teach that vocabulary only at ‘point of use’. Instead, preview relevant math vocabulary as a regular a part of the ‘background’ information that students receive in preparation to learn new math concepts or operations. (2) Model the relevant vocabulary when new concepts are taught. Strengthen students’ grasp of new vocabulary by reviewing a number of math problems with the class, each time consistently and explicitly modeling the use of appropriate vocabulary to describe the concepts being taught. Then have students engage in cooperative learning or individual practice activities in which they too must successfully use the new vocabulary—while the teacher provides targeted support to students as needed. (3) Ensure that students learn standard, widely accepted labels for common math terms and operations and that they use them consistently to describe their math problem-solving efforts.

**References**


Cover-Copy-Compare

Students who can be trusted to work independently and need extra drill and practice with math computational problems, spelling, or vocabulary words will benefit from Cover-Copy-Compare.

Preparing Cover-Copy-Compare Worksheets:
The teacher prepares worksheets for the student to use independently:

- For math worksheets, computation problems with answers appear on the left side of the sheet. The same computation problems appear on the right side of the page, unsolved. Here is a sample CCC item for math:

\[
\begin{array}{c}
88 \div 4312 \\
-352 \\
-792 \\
\hline
0
\end{array}
\]

- For spelling words, correctly spelled words are listed on the left of the page, with space on the right for the student to spell each word.

- For vocabulary items, words and their definitions are listed on the left side of the page, with space on the right for the student to write out each word and a corresponding definition for that word.

Using Cover-Copy-Compare Worksheets for Student Review:
When first introducing Cover-Copy-Compare worksheets to the student, the teacher gives the student an index card. The student is directed to look at each correct item (e.g., correctly spelled word, computation problem with solution) on the left side of the page.

- (For math problems.) The student is instructed to cover the correct model on the left side of the page with an index card and to copy the problem and compute the correct answer...
in the space on the right side of the sheet. The student then uncovers the correct answer on the left and checks his or her own work.

- (For spelling problems.) The student is instructed to cover the correct model on the left side of the page with an index card and to spell the word in the space on the right of the sheet. The student then uncovers the correct answer on the left to check his or her work.

- (For vocabulary items.) The student is instructed to cover the correct model on the left side of the page with an index card and to write both the word and its definition in the space on the right side of the sheet. The student then uncovers the correct model on the left to check his or her work.

Troubleshooting: How to Deal With Common Problems in Using 'Cover-Copy-Compare'

Q: How do I respond if the student simply copies the correct answers from the models into the answer blanks and tries to pass this off as his or her own work?

An essential requirement of Cover-Copy-Compare is that the student cover the correct model and attempt independently to solve the item using his or her own skills. If the student simply copies the correct answer from the model math problem or spelling word, the review process is short-circuited and the student will not benefit. If you suspect a student will copy rather than attempt to solve items on a CCC worksheet, arrange to have a peer tutor, adult in the classroom, or parent sit with the student to provide encouragement and monitoring.

Q: I have a student who is so disorganized that he will lose the index card before he can complete a CCC worksheet. Any suggestions?

Here is an idea for getting rid of that index card: You can fold the worksheet in half length-wise so that the answers appear on one side of the folded worksheet and the answer blanks appear on the other side. For each item, the student will peer at the correct model, then flip the folded sheet over to the right side to independently solve the item (with the correct model neatly folded out of sight).
Math Review: Promote Mastery of Math Facts Through Incremental Rehearsal

Incremental rehearsal builds student fluency in basic math facts (‘arithmetic combinations’) by pairing unknown computation items with a steadily increasing collection of known items. This intervention makes use of repeated, or massed, practice to promote fluency and guarantees that the student will experience a high rate of success.

Materials

- Index cards and pen

Steps to Implementing This Intervention

In preparation for this intervention:

1. The tutor first writes down on an index card in ink each math fact that a student is expected to master—but without the answer. NOTE: Educators can use the A-Plus Math Flashcard Creator, an on-line application, to make and print flashcards in addition, subtraction, multiplication, and division. The web address for the flashcard creator is: http://www.aplusmath.com/Flashcards/Flashcard_Creator.html

2. The tutor reviews the collection of math-fact cards with the student. Any of the math facts that the student can orally answer correctly within two seconds are considered to be known problems and are separated into one pile. Math facts that the student cannot yet answer correctly within two seconds are considered ‘unknown’ and collected in a second pile -- the ‘unknown facts’ deck.

3. The tutor next randomly selects 9 cards from the pile of known math facts and sets this subset of cards aside as the ‘known facts’ deck. The rest of the pile of cards containing known math facts is put away (‘discard deck’), not to be used further in this intervention.

During the intervention:

The tutor follows an incremental-rehearsal sequence each day when working with the student:

1. First, the tutor takes a single card from the ‘unknown facts’ deck. The tutor reads the math fact on the card aloud, provides the answer, and prompts the student to read off and answer the same unknown problem.

2. Next the tutor takes one math fact from the ‘known facts’ deck and pairs it with the unknown problem. When shown the two problems in sequence, the student is asked during the presentation of each math fact to read off the problem and answer it. The student is judged to be successful on a problem if he or she orally provides the correct answer to that problem within 2 seconds. If the student commits an error on any card or hesitates for longer than two seconds, the tutor reads the math fact on the card aloud, gives the answer, then prompts the
student to read off the same unknown problem and provide the answer. This review sequence continues until the student answers all cards within two seconds without errors.

3. The tutor then repeats the sequence—taking yet another problem from the ‘known facts’ deck to add to the expanding collection of math facts being reviewed (‘review deck’). Each time, the tutor prompts the student to read off and answer the whole series of math facts in the review deck, beginning with the unknown fact and then moving through the growing series of known facts that follow it.

4. When the review deck has expanded to include one ‘unknown’ math fact followed by nine ‘known’ math facts (a ratio of 90 percent ‘known’ material to 10 percent ‘unknown’ material), the last ‘known’ math fact that was added to the student’s review deck is discarded (put away with the ‘discard deck’). The previously ‘unknown’ math fact that the student has just successfully practiced in multiple trials is now treated as a ‘known’ math fact and is included as the first item in the nine-card ‘known facts’ deck for future drills.

5. The student is then presented with a new math fact to answer, taken from the ‘unknown facts’ deck. With each new ‘unknown’ math fact, the review sequence is again repeated as described above until the ‘unknown’ math fact is grouped incrementally with nine math facts from the ‘known facts’ deck—and on and on.

Daily review sessions are discontinued either when time runs out or when the student answers an ‘unknown’ math fact incorrectly three times.

Reference

Math Computation: Increase Accuracy By Intermixing Easy and Challenging Problems

Teachers can improve accuracy and positively influence the attitude of students when completing math-fact worksheets by intermixing ‘easy’ problems among the ‘challenging’ problems. Research shows that students are more motivated to complete computation worksheets when they contain some very easy problems interspersed among the more challenging items.

Materials

• Math computation worksheets & answer keys with a mixture of difficult and easy problems

Steps to Implementing This Intervention

1. The teacher first identifies one or more ‘challenging’ problem-types that are matched to the student’s current math-computation abilities (e.g., multiplying a 2-digit number by a 2-digit number with regrouping).

2. The teacher next identifies an ‘easy’ problem-type that the students can complete very quickly (e.g., adding or subtracting two 1-digit numbers).

3. The teacher then creates a series of student math computation worksheets with ‘easy’ computation problems interspersed at a fixed rate among the ‘challenging’ problems. (NOTE: Instructions are included below for creating interspersal worksheets using a free online application from www.interventioncentral.org.)

• If the student is expected to complete the worksheet independently as seat work or homework, ‘challenging’ and ‘easy’ problems should be interspersed at a 1:1 ratio (that is, every ‘challenging’ problem in the worksheet is followed by an ‘easy’ problem).

• If the student is to have the problems read aloud and then asked to solve the problems mentally and write down only the answer, the items should appear on the worksheet at a ratio of 3:1 (that is, every third ‘challenging’ problem is followed by an ‘easy’ one).

Directions for On-Line Creation of Worksheets With a Mix of Easy and Challenging Computation Problems (‘Interspersal Worksheets’)

By following the directions below, teachers can use a free on-line Math Worksheet Generator to create computation worksheets with easy problems interspersed among more challenging ones:

• The teacher goes to the following URL for the Math Worksheet Generator: http://www.interventioncentral.org/htmdocs/tools/mathprobe/allmult.php
• Displayed on that Math Worksheet Generator web page is a series of math computation goals for addition, subtraction, multiplication, and division. Teachers can select up to five different problem types to appear on a student worksheet. Each problem type is selected by clicking on the checkbox next to it.

• It is simple to create a worksheet with a 1:1 ratio of challenging and easy problems (that is, with an easy problem following every challenging problem). First, the teacher clicks the checkbox next to an ‘easy’ problem type that the student can compute very quickly (e.g., adding or subtracting two 1-digit numbers). Next the teacher selects a ‘challenging’ problem type that is instructionally appropriate for the student (e.g., multiplying a 2-digit number by a 2-digit number with regrouping). Then the teacher clicks the ‘Multiple Skill Computation Probe’ button. The computer program will then automatically create a student computation worksheet and teacher answer key with alternating easy and challenging problems.

• It is also no problem to create a worksheet with a higher (e.g., 2:1, 3:1, or 4:1) ratio of challenging problems to easy problems. The teacher first clicks the checkbox next to an ‘easy’ problem type that the student can compute very quickly (e.g., adding or subtracting two 1-digit numbers). The teacher then selects up to four different challenging problem types that are instructionally appropriate to the student. Depending on the number of challenging problem-types selected, when the teacher clicks the ‘Multiple Skill Computation Probe’ button, the computer program will create a student computation worksheet and teacher answer key that contain 2 (or 3 or 4) challenging problems for every easy problem.

Because the computer program generates new worksheets each time it is used, the teacher can enter the desired settings and—in one sitting—create and print off enough worksheets and answer keys to support a six- or eight-week intervention.

Reference

Applied Math Problems: Using Question-Answer Relationships (QARs) to Interpret Math Graphics

Students must be able to correctly interpret math graphics in order to correctly answer many applied math problems. Struggling learners in math often misread or misinterpret math graphics. For example, students may:

- overlook important details of the math graphic.
- treat irrelevant data on the math graphic as ‘relevant’.
- fail to pay close attention to the question before turning to the math graphic to find the answer
- not engage their prior knowledge both to extend the information on the math graphic and to act as a possible ‘reality check’ on the data that it presents.
- expect the answer to be displayed in plain sight on the math graphic, when in fact the graphic may require that readers first to interpret the data, then to plug the data into an equation to solve the problem.

Teachers need an instructional strategy to encourage students to be more savvy interpreters of graphics in applied math problems. One idea is to have them apply a reading comprehension strategy, Question-Answer Relationships (QARs) as a tool for analyzing math graphics. The four QAR question types (Raphael, 1982, 1986) are as follows:

- RIGHT THERE questions are fact-based and can be found in a single sentence, often accompanied by 'clue' words that also appear in the question.
- THINK AND SEARCH questions can be answered by information in the text--but require the scanning of text and the making of connections between disparate pieces of factual information found in different sections of the reading.
- AUTHOR AND YOU questions require that students take information or opinions that appear in the text and combine them with the reader's own experiences or opinions to formulate an answer.
- ON MY OWN questions are based on the students' own experiences and do not require knowledge of the text to answer.

Steps to Implementing This Intervention

Teachers use a 4-step instructional sequence to teach students to use Question-Answer Relationships (QARs) to better interpret math graphics:

1. **Step 1: Distinguishing Among Different Kinds of Graphics**

   Students are first taught to differentiate between five common types of math graphics: table (grid with information contained in cells), chart (boxes with possible connecting lines or arrows), picture (figure with labels), line graph, bar graph.

   Students note significant differences between the various types of graphics, while the teacher
records those observations on a wall chart. Next students are shown examples of graphics and directed to identify the general graphic type (table, chart, picture, line graph, bar graph) that each sample represents.

As homework, students are assigned to go on a ‘graphics hunt’, locating graphics in magazines and newspapers, labeling them, and bringing them to class to review.

2. Interpreting Information in Graphics

Over several instructional sessions, students learn to interpret information contained in various types of math graphics. For these activities, students are paired off, with stronger students matched with less strong ones.

The teacher sets aside a separate session to introduce each of the graphics categories. The presentation sequence is ordered so that students begin with examples of the most concrete graphics and move toward the more abstract. The graphics sequence in order of increasing difficulty is: Pictures > tables > bar graphs > charts > line graphs.

At each session, student pairs examine examples of graphics from the category being explored that day and discuss questions such as: “What information does this graphic present? What are strengths of this type of graphic for presenting data? What are possible weaknesses?” Student pairs record their findings and share them with the large group at the end of the session.

3. Linking the Use of Question-Answer Relations (QARs) to Graphics

In advance of this lesson, the teacher prepares a series of data questions and correct answers. Each question and answer is paired with a math graphic that contains information essential for finding the answer.

At the start of the lesson, students are each given a set of 4 index cards with titles and descriptions of each of the 4 QAR questions: RIGHT THERE, THINK AND SEARCH, AUTHOR AND YOU, ON MY OWN. (TMEASAVING TIP: Students can create their own copies of these QAR review cards as an in-class activity.)

Working first in small groups and then individually, students read each teacher-prepared question, study the matching graphic, and ‘verify’ the provided answer as correct. They then identify the type of question being posed in that applied problem, using their QAR index cards as a reference.

4. Using Question-Answer Relationships (QARs) Independently to Interpret Math Graphics

Students are now ready to use the QAR strategy independently to interpret graphics. They are given a laminated card as a reference with 6 steps to follow whenever they attempt to solve an
applied problem that includes a math graphic:

- Read the question,
- Review the graphic,
- Reread the question,
- Choose a Question-Answer Relationship that matches the question in the applied problem
- Answer the question, and
- Locate the answer derived from the graphic in the answer choices offered.

Students are strongly encouraged NOT to read the answer choices offered on a multiple-choice item until they have first derived their own answer—to prevent those choices from short-circuiting their inquiry.

References


Math Computation: Increase Accuracy and Productivity Rates Via Self-Monitoring and Performance Feedback

Students can improve both their accuracy and fluency on math computation worksheets by independently self-monitoring their computation speed, charting their daily progress, and earning rewards for improved performance.

Materials

- Collection of student math computation worksheets & matching answer keys (NOTE: Educators can use a free online application to create math computation worksheets and answer keys at http://www.interventioncentral.org/htmdocs/tools/mathprobe/addsing.php)

- Student self-monitoring chart

Steps to Implementing This Intervention

In preparation for this intervention:

- the teacher selects one or more computation problem types that the student needs to practice. Using that set of problem types as a guide, the teacher creates a number of standardized worksheets with similar items to be used across multiple instructional days. (A Math Worksheet Generator that will create these worksheets automatically can be accessed at http://www.interventioncentral.org).

- the teacher prepares a progress-monitoring chart. The vertical axis of the chart extends from 0 to 100 and is labeled ‘Correct Digits’. The horizontal axis of the chart is labeled ‘Date’.

- the teacher creates a menu of rewards that the student can choose from on a given day if the student was able to exceed his or her previously posted computation fluency score.

At the start of the intervention, the teacher meets with the student. The teacher shows the student a sample math computation worksheet and answer key. The teacher tells the student that the student will have the opportunity to complete similar math worksheets as time drills and chart the results. The student is told that he or she will win a reward on any day when the student’s number of correctly computed digits on the worksheet exceeds that of the previous day.

During each day of the intervention:

1. The student is given one of the math computation worksheets previously created by the teacher, along with an answer key. The student first consults his or her progress-monitoring chart and notes the most recent charted computation fluency score previously posted. The student is encouraged to try to exceed that score.
2. When the intervention session starts, the student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems on the computation worksheet as possible. The student sets a timer for the allocated time and works on the computation sheet until the timer rings.

3. The student then uses the answer key to check his or her work, giving credit for each correct digit in an answer. (A 'correct digits' is defined as a digit of the correct value that appears in the correct place-value location in an answer. In this scoring method, students can get partial credit even if some of the digits in an answer are correct and some are incorrect.).

4. The student plots his or her computational fluency score on the progress-monitoring chart and writes the current date at the bottom of the chart below the plotted data point. The student is allowed to select a choice from the reward menu if he or she exceeds his or her most recent, previously posted fluency score.

References


Building Blocks of Effective Instruction

Good classroom instruction is no accident. Two powerful tools for analyzing the quality of student instruction are the Instructional Hierarchy and the Learn Unit.

**Instructional Hierarchy.** As students are taught new academic skills, they go through a series of predictable learning stages. At the start, a student is usually halting and uncertain as he or she tries to use the target skill. With teacher feedback and lots of practice, the student becomes more fluent, accurate, and confident in using the skill. It can be very useful to think of these phases of learning as a hierarchy (See chart on page 2). The learning hierarchy (Haring, Lovitt, Eaton, & Hansen, 1978) has four stages: acquisition, fluency, generalization, and adaptation:

1. **Acquisition.** The student has begun to learn how to complete the target skill correctly but is not yet accurate or fluent in the skill. The goal in this phase is to improve accuracy.
2. **Fluency.** The student is able to complete the target skill accurately but works slowly. The goal of this phase is to increase the student’s speed of responding (fluency).
3. **Generalization.** The student is accurate and fluent in using the target skill but does not typically use it in different situations or settings. Or the student may confuse the target skill with ‘similar’ skills. The goal of this phase is to get the student to use the skill in the widest possible range of settings and situations, or to accurately discriminate between the target skill and ‘similar’ skills.
4. **Adaptation.** The student is accurate and fluent in using the skill. He or she also uses the skill in many situations or settings. However, the student is not yet able to modify or adapt the skill to fit novel task-demands or situations.

The ‘Learn Unit’. At the core of good instruction lies the “Learn Unit”, a 3-step process in which the student is invited to engage in an academic task, delivers a response, and then receives immediate feedback about how he or she did on the task (Heward, 1996). Here is an explanation of the stages of the ‘Learn Unit’:

1. **Academic Opportunity to Respond.** The student is presented with a meaningful opportunity to respond to an academic task. A question posed by the teacher, a math word problem, and a spelling item on an educational computer ‘Word Gobbler’ game could all be considered academic opportunities to respond.
2. **Active Student Response.** The student answers the item, solves the problem presented, or completes the academic task. Answering the teacher’s question, computing the answer to a math word problem (and showing all work), and typing in the correct spelling of an item when playing an educational computer game are all examples of active student responding.
3. **Performance Feedback.** The student receives timely feedback about whether his or her response is correct—often with praise and encouragement. A teacher exclaiming ‘Right! Good job!’ when a student gives a response in class, a student using an answer key to check her answer to a math word problem, and a computer message that says ‘Congratulations! You get 2 points for correctly spelling this word!’ are all examples of corrective feedback.

The more frequently a student cycles through complete ‘Learn Unit’ trials, the faster that student is likely to make learning progress. If any one of these steps is missing, the quality of instruction will probably be compromised.

**References**


Instructional Hierarchy: Matching Interventions to Student Learning Stage (Haring, et al., 1978)

<table>
<thead>
<tr>
<th>Learning Stage</th>
<th>Student ‘Look-Fors’…</th>
<th>What strategies are effective…</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Acquisition:</strong> Exit Goal: The student can perform the skill accurately with little adult support.</td>
<td>• Is just beginning to learn skill&lt;br&gt;• Not yet able to perform learning task reliably or with high level of accuracy</td>
<td>• Teacher actively demonstrates target skill&lt;br&gt;• Teacher uses ‘think-aloud’ strategy-- especially for thinking skills that are otherwise covert&lt;br&gt;• Student has models of correct performance to consult as needed (e.g., correctly completed math problems on board)&lt;br&gt;• Student gets feedback about correct performance&lt;br&gt;• Student receives praise, encouragement for effort</td>
</tr>
<tr>
<td><strong>Fluency:</strong> Exit Goals: The student (a) has learned skill well enough to retain (b) has learned skill well enough to combine with other skills, (c) is as fluent as peers.</td>
<td>• Gives accurate responses to learning task&lt;br&gt;• Performs learning task slowly, haltingly</td>
<td>• Teacher structures learning activities to give student opportunity for active (observable) responding&lt;br&gt;• Student has frequent opportunities to drill (direct repetition of target skill) and practice (blending target skill with other skills to solve problems)&lt;br&gt;• Student gets feedback on fluency and accuracy of performance&lt;br&gt;• Student receives praise, encouragement for increased fluency</td>
</tr>
<tr>
<td><strong>Generalization:</strong> Exit Goals: The student (a) uses the skill across settings, situations; (b) does not confuse target skill with similar skills</td>
<td>• Is accurate and fluent in responding&lt;br&gt;• May fail to apply skill to new situations, settings&lt;br&gt;• May confuse target skill with similar skills (e.g., confusing ‘+’ and ‘x’ number operation signs)</td>
<td>• Teacher structures academic tasks to require that the student use the target skill regularly in assignments.&lt;br&gt;• Student receives encouragement, praise, reinforcers for using skill in new settings, situations&lt;br&gt;• If student confuses target skill with similar skill(s), the student is given practice items that force him/her to correctly discriminate between similar skills&lt;br&gt;• Teacher works with parents to identify tasks that the student can do outside of school to practice target skill&lt;br&gt;• Student gets periodic opportunities to review, practice target skill to ensure maintenance</td>
</tr>
<tr>
<td><strong>Adaptation:</strong> Exit Goal: The Adaptation phase is continuous and has no exit criteria.</td>
<td>• Is fluent and accurate in skill&lt;br&gt;• Applies skill in novel situations, settings without prompting&lt;br&gt;• Does not yet modify skill as needed to fit new situations (e.g., child says ‘Thank you’ in all situations, does not use modified, equivalent phrases such as “I appreciate your help.”)</td>
<td>• Teacher helps student to articulate the ‘big ideas’ or core element(s) of target skill that the student can modify to face novel tasks, situations (e.g., fractions, ratios, and percentages link to the ‘big idea’ of the part in relation to the whole; ‘Thank you’ is part of a larger class of polite speech)&lt;br&gt;• Train for adaptation: Student gets opportunities to practice the target skill with modest modifications in new situations, settings with encouragement, corrective feedback, praise, other reinforcers.&lt;br&gt;• Encourage student to set own goals for adapting skill to new and challenging situations.</td>
</tr>
</tbody>
</table>