RTI: Best Practices in Mathematics Interventions: Grades K-5

Jim Wright
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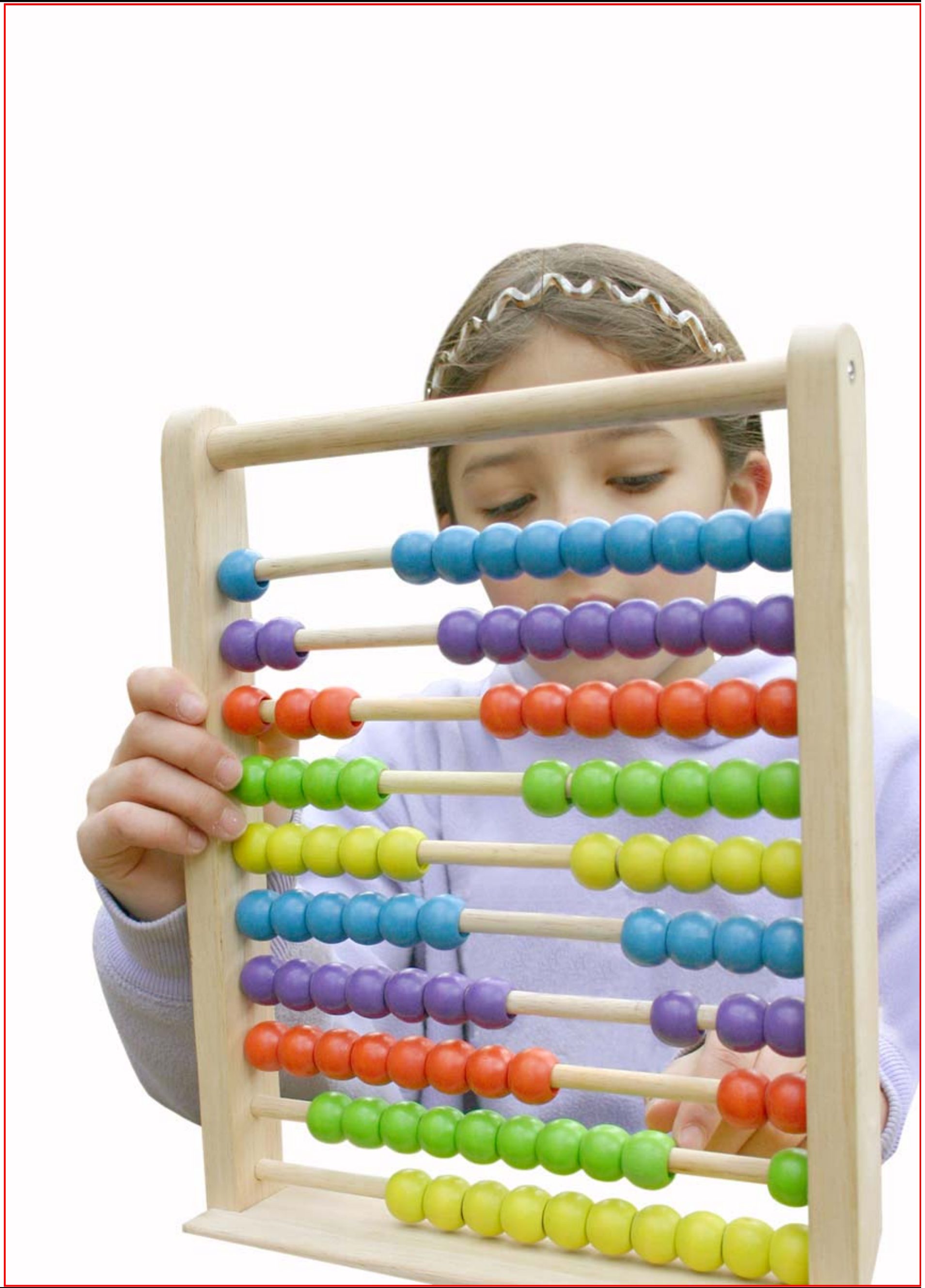


Workshop PPTs and Handout Available at:

http://www.interventioncentral.org/swgaresa

Workshop Agenda: RTI Challenges…

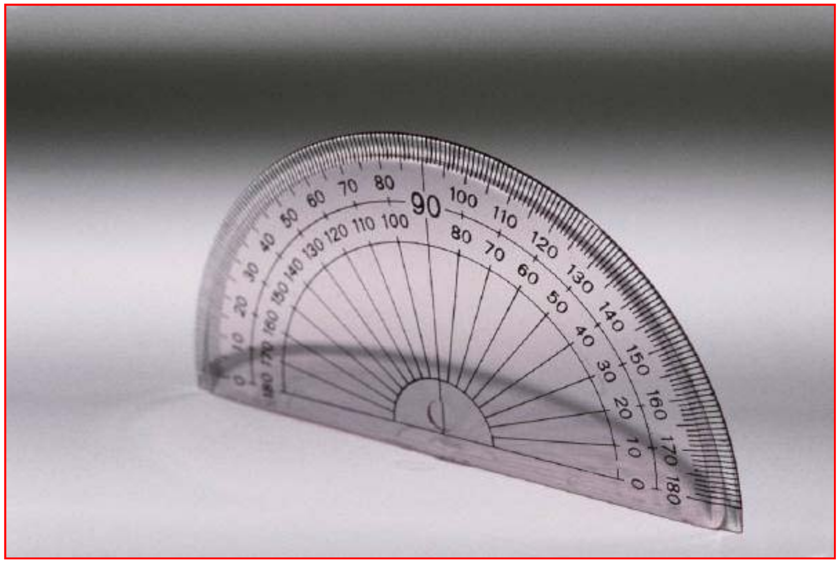
- Defining Research-Based Principles of Effective Math Instruction & Intervention
- Understanding the Student With 'Math Difficulties'
- Finding Effective, Research-Based Math Interventions
- Screening and Progress-Monitoring for Students With Math Difficulties
- Finding Web Resources to Support Math Assessment & Interventions

'Elbow Group' Activity: What are common student mathematics concerns in your school?

In your 'elbow groups':

- Discuss the most common student mathematics problems that you encounter in your school(s). At what grade level do you typically encounter these problems?
- Be prepared to share your discussion points with the larger group.

RTI Challenge: Defining Research-Based Principles of Effective Math Instruction & Intervention



An RTI Challenge: Limited Research to Support Evidence-Based Math Interventions

"… in contrast to reading, core math programs that are supported by research, or that have been constructed according to clear research-based principles, are not easy to identify. Not only have exemplary core programs not been identified, but also there are no tools available that we know of that will help schools analyze core math programs to determine their alignment with clear research-based principles." p. 459

Source: Clarke, B., Baker, S., & Chard, D. (2008). Best practices in mathematics assessment and intervention with elementary students. In A. Thomas & J. Grimes (Eds.), Best practices in school psychology V (pp. 453-463).

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BOARDS & COMMISSIONS

National Mathematics Advisory Panel

National Mathematics Advisory Panel Releases Final Report

On March 13, 2008, the National Mathematics Advisory Panel presented its Final Report to the President of the United States and the Secretary of Education. Copies of these ground-breaking reports, rich with information for parents, teachers, policy makers, the research community, and others, are provided below.

Foundations for Success: Report of the National Mathematics Advisory Panel

Final Report PDF (851 KB) | Word (1 MB)

Draft Task Group Reports

- Conceptual Knowledge and Skills Word (1.3 MB)
- Learning Processes Word (7.9 MB)
- Instructional Practices Word (2.9 MB)
- Teachers Word (1.2 MB)
- Assessment Word (876 KB)

Draft Subcommittee Reports

- Standards of Evidence PDF (68 KB) | Word (276 KB)
- Instructional Materials Word (958 KB)
- National Survey of Algebra Teachers for the National Math Panel PDF (4.1 MB) | Word (3.2 MB)

Fact Sheet

Paper copies of these reports may be ordered at EDPubs.ed.gov.

If you need any of these documents in an alternative format, please contact the National Math Panel at NationalMathPanel@ed.gov.

NoChild LeftBehind

Related Topics:

- No Related Topics Found

National Mathematics Advisory Panel Report
13 March 2008

Math Advisory Panel Report at:

http://www.ed.gov/mathpanel

2008 National Math Advisory Panel Report: Recommendations

- "The areas to be studied in mathematics from pre-kindergarten through eighth grade should be streamlined and a well-defined set of the most important topics should be emphasized in the early grades. Any approach that revisits topics year after year without bringing them to closure should be avoided."
- "Proficiency with whole numbers, fractions, and certain aspects of geometry and measurement are the foundations for algebra. Of these, knowledge of fractions is the most important foundational skill not developed among American students."
- "Conceptual understanding, computational and procedural fluency, and problem solving skills are equally important and mutually reinforce each other. Debates regarding the relative importance of each of these components of mathematics are misguided."
- "Students should develop immediate recall of arithmetic facts to free the "working memory" for solving more complex problems."

Source: National Math Panel Fact Sheet. (March 2008). Retrieved on March 14, 2008, from http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-factsheet.html

The Elements of Mathematical Proficiency: What the Experts Say…

5 Strands of Mathematical Proficiency

1. Understanding
2. Computing
3. Applying
4. Reasoning
5. Engagement

Source: *National Research Council. (2002). Helping children learn mathematics. Mathematics Learning Study Committee, J. Kilpatrick & J. Swafford, Editors, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.*

5 Big Ideas in Beginning Reading

1. Phonemic Awareness
2. Alphabetic Principle
3. Fluency with Text
4. Vocabulary
5. Comprehension

Source: Big ideas in beginning reading. University of Oregon. Retrieved September 23, 2007, from http://reading.uoregon.edu/index.php

Five Strands of Mathematical Proficiency

1. ***Understanding:*** *Comprehending mathematical concepts, operations, and relations--knowing what mathematical symbols, diagrams, and procedures mean.*

2. ***Computing:*** *Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.*

3. ***Applying:*** *Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.*

Source: *National Research Council. (2002). Helping children learn mathematics. Mathematics Learning Study Committee, J. Kilpatrick & J. Swafford, Editors, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.*

Five Strands of Mathematical Proficiency (Cont.)

4. ***Reasoning:*** *Using logic to explain and justify a solution to a problem or to extend from something known to something less known.*

5. ***Engaging:*** *Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.*

Source: *National Research Council. (2002). Helping children learn mathematics. Mathematics Learning Study Committee, J. Kilpatrick & J. Swafford, Editors, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.*

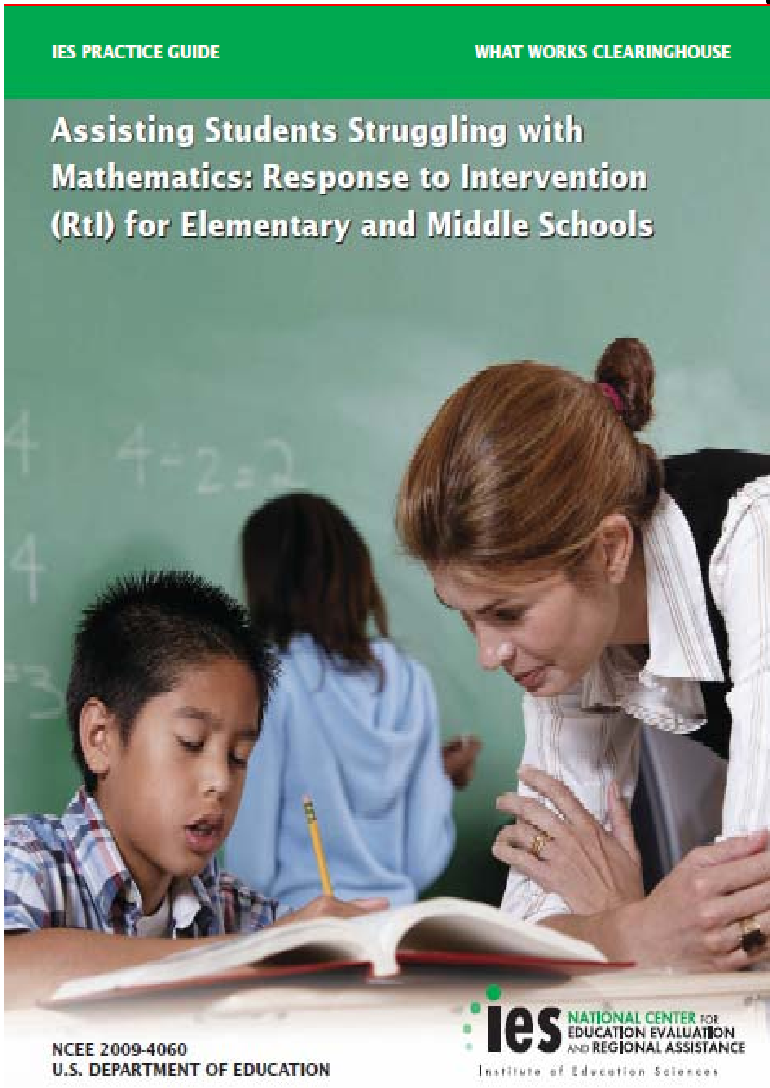
Table Activity: Evaluate Your School's Math Proficiency…

- *As a group, review the National Research Council 'Strands of Math Proficiency'.*
- *Which strand do you feel that your school / curriculum does the **best** job of helping students to attain proficiency?*
- *Which strand do you feel that your school / curriculum should put **the greatest effort** to figure out how to help students to attain proficiency?*
- *Be prepared to share your results.*

Five Strands of Mathematical Proficiency (NRC, 2002)

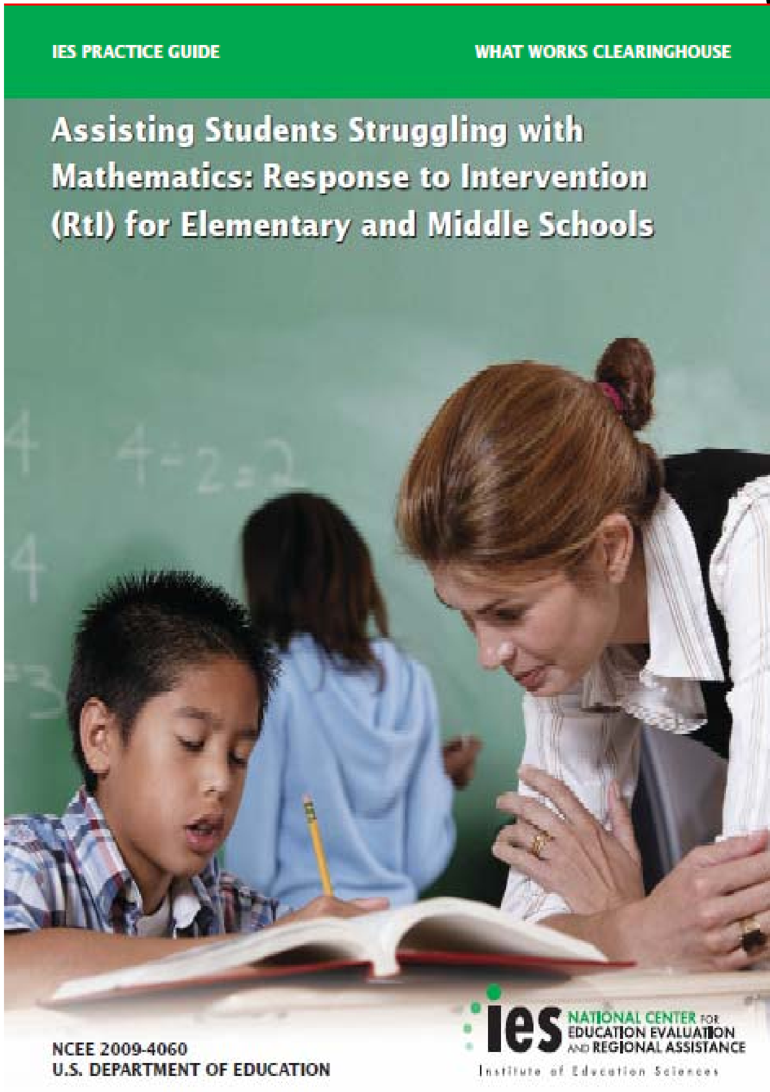
1. ***Understanding:*** *Comprehending mathematical concepts, operations, and relations--knowing what mathematical symbols, diagrams, and procedures mean.*
2. ***Computing:*** *Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.*
3. ***Applying:*** *Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.*
4. ***Reasoning:*** *Using logic to explain and justify a solution to a problem or to extend from something known to something less known.*
5. ***Engaging:*** *Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.*

Assisting Students Struggling with Mathematics: RtI for Elementary & Middle Schools: 8 Recommendations



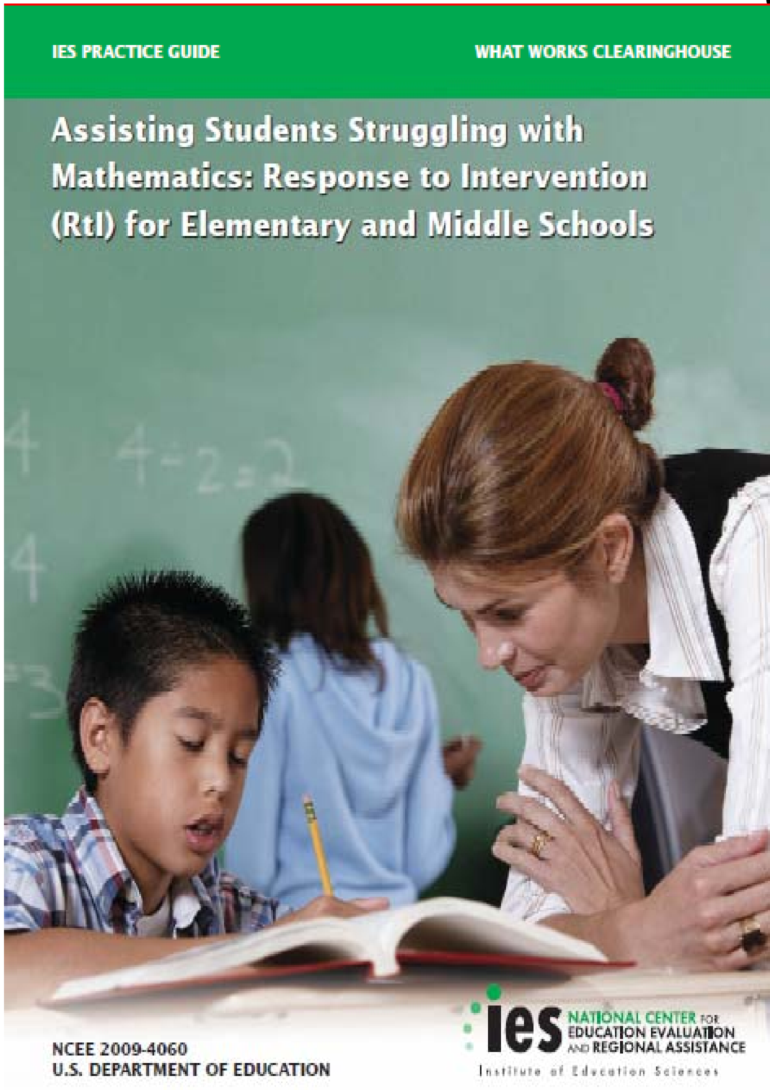
- **Recommendation 1.** Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk
- **Recommendation 2.** Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.

Assisting Students Struggling with Mathematics: RtI for Elementary & Middle Schools: 8 Recommendations (Cont.)



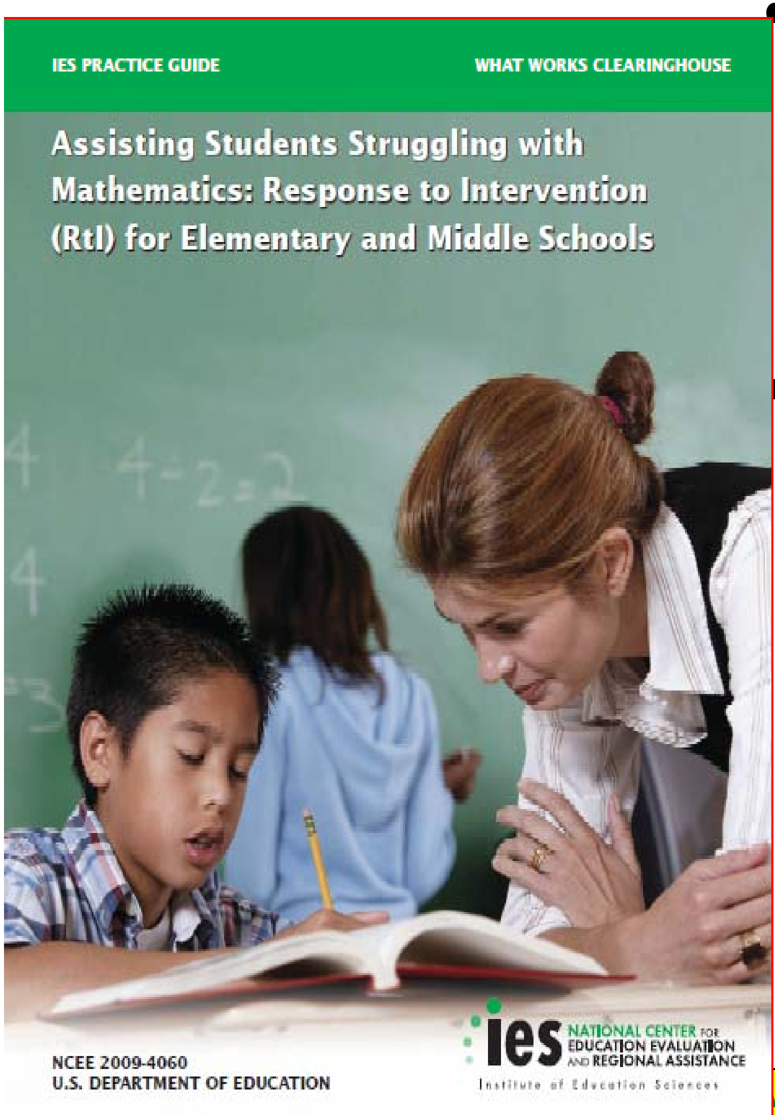
- **Recommendation 3.** Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review
- **Recommendation 4.** Interventions should include instruction on solving word problems that is based on common underlying structures.

Assisting Students Struggling with Mathematics: RtI for Elementary & Middle Schools: 8 Recommendations (Cont.)



- **Recommendation 5.** Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas
- **Recommendation 6.** Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts

Assisting Students Struggling with Mathematics: RtI for Elementary & Middle Schools: 8 Recommendations (Cont.)



- **Recommendation 7.** Monitor the progress of students receiving supplemental instruction and other students who are at risk
- **Recommendation 8.** Include motivational strategies in tier 2 and tier 3 interventions.

RTI Interventions: What If There is No Commercial Intervention Package or Program Available?

"Although commercially prepared programs and the subsequent manuals and materials are inviting, they are not necessary. … A recent review of research suggests that interventions are research based and likely to be successful, if they are correctly targeted and provide explicit instruction in the skill, an appropriate level of challenge, sufficient opportunities to respond to and practice the skill, and immediate feedback on performance…Thus, these [elements] could be used as criteria with which to judge potential …interventions." p. 88

Source: Burns, M. K., & Gibbons, K. A. (2008). *Implementing response-to-intervention in elementary and secondary schools*. Routledge: New York.

Finding the Right Spark: Strategies for Motivating the Resistant Learner
(Excerpt)

Motivation Deficit 1: *The student is unmotivated because he or she cannot do the assigned work.*

- **Profile of a Student with This Motivation Problem:** The student lacks essential skills required to do the task.

Motivation Deficit 1: Cannot Do the Work

- **Profile of a Student with This Motivation Problem (Cont.):** Areas of deficit might include:
- *Basic academic skills.* Basic skills have straightforward criteria for correct performance (e.g., the student defines vocabulary words or decodes text or computes 'math facts') and comprise the building-blocks of more complex academic tasks (Rupley, Blair, & Nichols, 2009).
- *Cognitive strategies.* Students employ specific cognitive strategies as "guiding procedures" to complete more complex academic tasks such as reading comprehension or writing (Rosenshine, 1995
- *Academic-enabling skills.* Skills that are 'academic enablers' (DiPerna, 2006) are not tied to specific academic knowledge but rather aid student learning across a wide range of settings and tasks (e.g., organizing work materials, time management).

Motivation Deficit 1: Cannot Do the Work (Cont.)

- **What the Research Says:** When a student lacks the capability to complete an academic task because of limited or missing basic skills, cognitive strategies, or academic-enabling skills, that student is still in the acquisition stage of learning (Haring et al., 1978). That student cannot be expected to be motivated or to be successful as a learner unless he or she is first explicitly taught these weak or absent essential skills (Daly, Witt, Martens & Dool, 1997).

Motivation Deficit 1: Cannot Do the Work (Cont.)

- How to Verify the Presence of This Motivation Problem: The teacher collects information (e.g., through observations of the student engaging in academic tasks; interviews with the student; examination of work products, quizzes, or tests) demonstrating that the student lacks basic skills, cognitive strategies, or academic-enabling skills essential to the academic task.

Motivation Deficit 1: Cannot Do the Work (Cont.)

- **How to Fix This Motivation Problem:** Students who are not motivated because they lack essential skills need to be taught those skills.

  Direct-Instruction Format. Students learning new material, concepts, or skills benefit from a ‘direct instruction’ approach. (Burns, VanDerHeyden & Boice, 2008; Rosenshine, 1995; Rupley, Blair, & Nichols, 2009).

Motivation Deficit 1: Cannot Do the Work (Cont.)

- **How to Fix This Motivation Problem:** When following a direct-instruction format, the teacher:
  - ❑ensures that the lesson content is appropriately matched to students' abilities.
  - ❑opens the lesson with a brief review of concepts or material that were previously presented.
  - ❑states the goals of the current day's lesson.
  - ❑breaks new material into small, manageable increments, or steps.

Motivation Deficit 1: Cannot Do the Work (Cont.)

- **How to Fix This Motivation Problem:** When following a direct-instruction format, the teacher:
  - ❑ throughout the lesson, provides adequate explanations and detailed instructions for all concepts and materials being taught. NOTE: Verbal explanations can include ‘talk-alouds’ (e.g., the teacher describes and explains each step of a cognitive strategy) and ‘think-alouds’ (e.g., the teacher applies a cognitive strategy to a particular problem or task and verbalizes the steps in applying the strategy).
  - ❑ regularly checks for student understanding by posing frequent questions and eliciting group responses.

Motivation Deficit 1: Cannot Do the Work (Cont.)

- **How to Fix This Motivation Problem:** When following a direct-instruction format, the teacher:
  - ❑verifies that students are experiencing sufficient success in the lesson content to shape their learning in the desired direction and to maintain student motivation and engagement.
  - ❑provides timely and regular performance feedback and corrections throughout the lesson as needed to guide student learning.

Motivation Deficit 1: Cannot Do the Work (Cont.)

- **How to Fix This Motivation Problem:** When following a direct-instruction format, the teacher:
  - ❑ allows students the chance to engage in practice activities distributed throughout the lesson (e.g., through teacher demonstration; then group practice with teacher supervision and feedback; then independent, individual student practice).
  - ❑ ensures that students have adequate support (e.g., clear and explicit instructions; teacher monitoring) to be successful during independent seatwork practice activities.

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Activity: Essential Elements of Direct Instruction

- Read through the essential elements of direct instruction that appear on p. 8 of your packet.
- Identify the element(s) that you believe would present the greatest challenge to implement on a regular basis.
- Brainstorm ideas to overcome these challenges.

Direct-Instruction Format. Students learning new material, concepts, or skills benefit from a 'direct instruction' approach. (Burns, VanDerHeyden & Boice, 2008; Rosenshine, 1995; Rupley, Blair, & Nichols, 2009). When following a direct-instruction format, the teacher:

- ❑ ensures that the lesson content is appropriately matched to students' abilities.
- ❑ opens the lesson with a brief review of concepts or material that were previously presented.
- ❑ states the goals of the current day's lesson.
- ❑ breaks new material into small, manageable increments, or steps.
- ❑ throughout the lesson, provides adequate explanations and detailed instructions for all concepts and materials being taught. NOTE: Verbal explanations can include 'talk-alouds' (e.g., the teacher describes and explains each step of a cognitive strategy) and 'think-alouds' (e.g., the teacher applies a cognitive strategy to a particular problem or task and verbalizes the steps in applying the strategy).
- ❑ regularly checks for student understanding by posing frequent questions and eliciting group responses.
- ❑ verifies that students are experiencing sufficient success in the lesson content to shape their learning in the desired direction and to maintain student motivation and engagement.
- ❑ provides timely and regular performance feedback and corrections throughout the lesson as needed to guide student learning.
- ❑ allows students the chance to engage in practice activities distributed throughout the lesson (e.g., through teacher demonstration; then group practice with teacher supervision and feedback; then independent, individual student practice).
- ❑ ensures that students have adequate support (e.g., clear and explicit instructions; teacher monitoring) to be successful during independent seatwork practice activities.

Teaching Math Vocabulary

Vocabulary: Why This Instructional Goal is Important

As vocabulary terms become more specialized in content area courses, students are less able to derive the meaning of unfamiliar words from context alone.

Students must instead learn vocabulary through more direct means, including having opportunities to explicitly memorize words and their definitions.

Students may require 12 to 17 meaningful exposures to a word to learn it.

Comprehending Math Vocabulary: The Barrier of Abstraction

"…when it comes to abstract mathematical concepts, words describe activities or relationships that often lack a visual counterpart. Yet studies show that children grasp the idea of quantity, as well as other relational concepts, from a very early age…. As children develop their capacity for understanding, language, and its vocabulary, becomes a vital cognitive link between a child's natural sense of number and order and conceptual learning. "

-Chard, D. (n.d.)

Source: Chard, D. (n.d.. *Vocabulary strategies for the mathematics classroom*. Retrieved November 23, 2007, from http://www.eduplace.com/state/pdf/author/chard_hmm05.pdf.

Math Vocabulary: Classroom (Tier I) Recommendations

- *Preteach math vocabulary*. Math vocabulary provides students with the language tools to grasp abstract mathematical concepts and to explain their own reasoning. Therefore, do not wait to teach that vocabulary only at 'point of use'. Instead, preview relevant math vocabulary as a regular a part of the 'background' information that students receive in preparation to learn new math concepts or operations.
- *Model the relevant vocabulary when new concepts are taught*. Strengthen students' grasp of new vocabulary by reviewing a number of math problems with the class, each time consistently and explicitly modeling the use of appropriate vocabulary to describe the concepts being taught. Then have students engage in cooperative learning or individual practice activities in which they too must successfully use the new vocabulary—while the teacher provides targeted support to students as needed.
- *Ensure that students learn standard, widely accepted labels* for common math terms and operations and that they use them consistently to describe their math problem-solving efforts.

Source: Chard, D. (n.d.. *Vocabulary strategies for the mathematics classroom*. Retrieved November 23, 2007, from http://www.eduplace.com/state/pdf/author/chard_hmm05.pdf.

Promoting Math Vocabulary: Other Guidelines

- Create a standard list of math vocabulary for each grade level (elementary) or course/subject area (for example, geometry).
- Periodically check students' mastery of math vocabulary (e.g., through quizzes, math journals, guided discussion, etc.).
- Assist students in learning new math vocabulary by first assessing their previous knowledge of vocabulary terms (e.g., *protractor*; *product*) and then using that past knowledge to build an understanding of the term.
- For particular assignments, have students identify math vocabulary that they don't understand. In a cooperative learning activity, have students discuss the terms. Then review any remaining vocabulary questions with the entire class.
- Encourage students to use a math dictionary in their vocabulary work.
- Make vocabulary a central part of instruction, curriculum, and assessment—rather than treating as an afterthought.

Source: Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786-795.

Math Instruction: Unlock the Thoughts of Reluctant Students Through Class Journaling

Students can effectively clarify their knowledge of math concepts and problem-solving strategies through regular use of class ‘math journals’.

- At the start of the year, the teacher introduces the journaling weekly assignment in which students respond to teacher questions.
- At first, the teacher presents ‘safe’ questions that tap into the students’ opinions and attitudes about mathematics (e.g., ‘How important do you think it is nowadays for cashiers in fast-food restaurants to be able to calculate in their head the amount of change to give a customer?”). As students become comfortable with the journaling activity, the teacher starts to pose questions about the students’ own mathematical thinking relating to specific assignments. Students are encouraged to use numerals, mathematical symbols, and diagrams in their journal entries to enhance their explanations.
- The teacher provides brief written comments on individual student entries, as well as periodic oral feedback and encouragement to the entire class.
- Teachers will find that journal entries are a concrete method for monitoring student understanding of more abstract math concepts. To promote the quality of journal entries, the teacher might also assign them an effort grade that will be calculated into quarterly math report card grades.

Source: *Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students. Learning Disabilities Research & Practice, 20(2), 119–135.*

Teaching Math Symbols

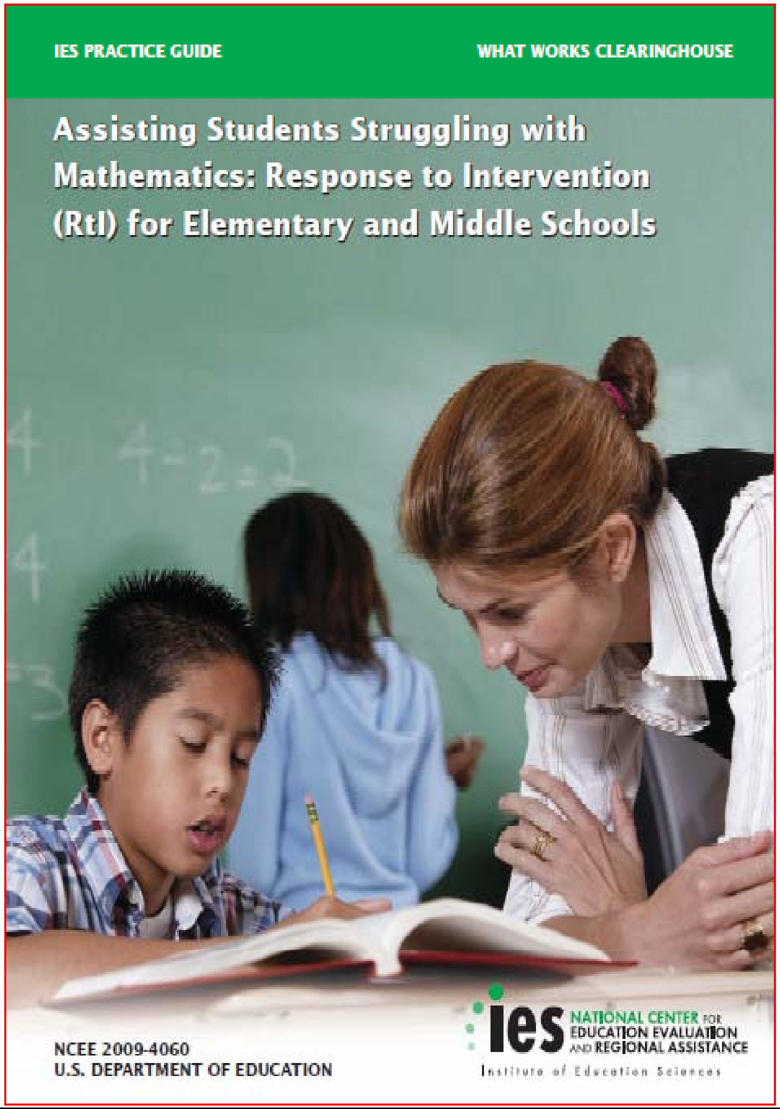
Learning Math Symbols: 3 Card Games

1. The interventionist writes math symbols that the student is to learn on index cards. The names of those math symbols are written on separate cards. The cards can then be used for students to play matching games or to attempt to draw cards to get a pair.
2. Create a card deck containing math symbols or their word equivalents. Students take turns drawing cards from the deck. If they can use the symbol/word on the selected card to formulate a correct 'mathematical sentence', the student wins the card. For example, if the student draws a card with the term 'negative number' and says that "*A negative number is a real number that is less than 0*", the student wins the card.
3. Create a deck containing math symbols and a series of numbers appropriate to the grade level. Students take turns drawing cards. The goral is for the student to lay down a series of cards to form a math expression. If the student correctly solves the expression, he or she earns a point for every card laid down.

Source: Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786-795.

Use Visual Representations in Math Problem-Solving

Assisting Students Struggling with Mathematics: RtI for Elementary & Middle Schools



Encourage Students to Use Visual Representations to Enhance Understanding of Math Reasoning

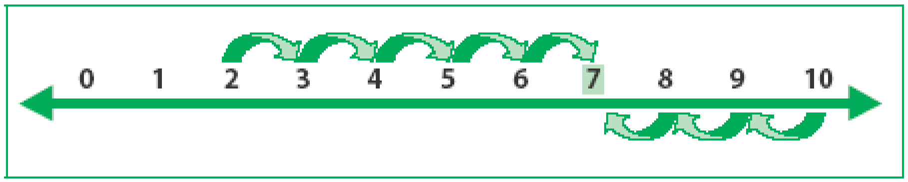
- Students should be taught to use standard visual representations in their math problem solving (e.g., numberlines, arrays, etc.)
- Visual representations should be explicitly linked with “the standard symbolic representations used in mathematics” p. 31
- Concrete manipulatives *can* be used, but only if visual representations are too abstract for student needs.

Concrete Manipulatives>>>
Visual Representations>>>
Representation Through Math Symbols

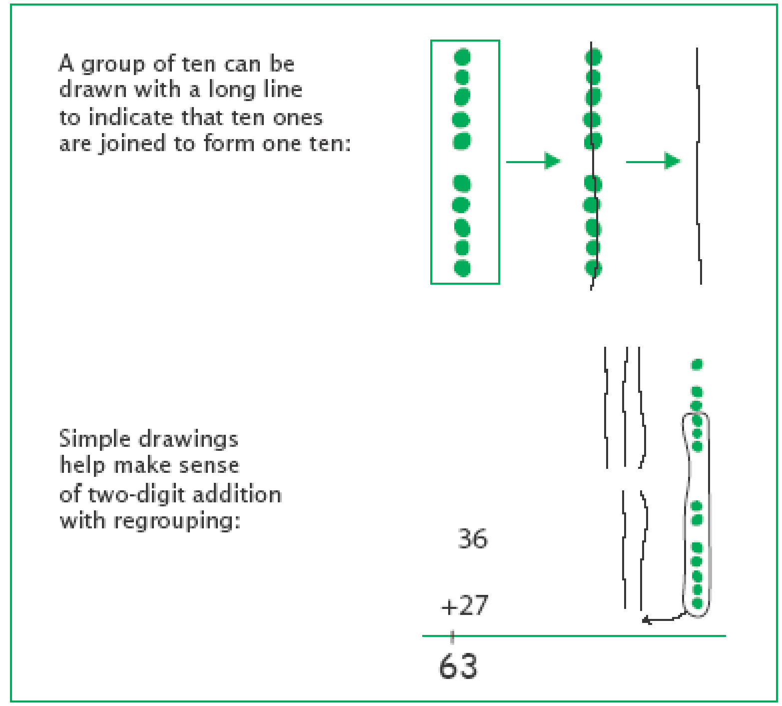
Source: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel,B. (2009). Assisting students struggling with mathematics: Response to Intervention Rtl) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/.

Examples of Math Visual Representations

Example 4. Representation of the counting on strategy using a number line



Example 5. Using visual representations for multidigit addition



Source: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to Intervention RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/.

Schools Should Build Their Capacity to Use Visual Representations in Math

Caution: Many intervention materials offer only limited guidance and examples in use of visual representations to promote student learning in math.

Therefore, schools should increase their capacity to coach interventionists in the more extensive use of visual representations. For example, a school might match various types of visual representation formats to key objectives in the math curriculum.

Source: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel,B. (2009). Assisting students struggling with mathematics: Response to Intervention Rtl) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/.

Teach Students to Identify Underlying Structures of Math Problems

Teach Students to Identify 'Underlying Structures' of Word Problems

Students should be taught to classify specific problems into problem-types:

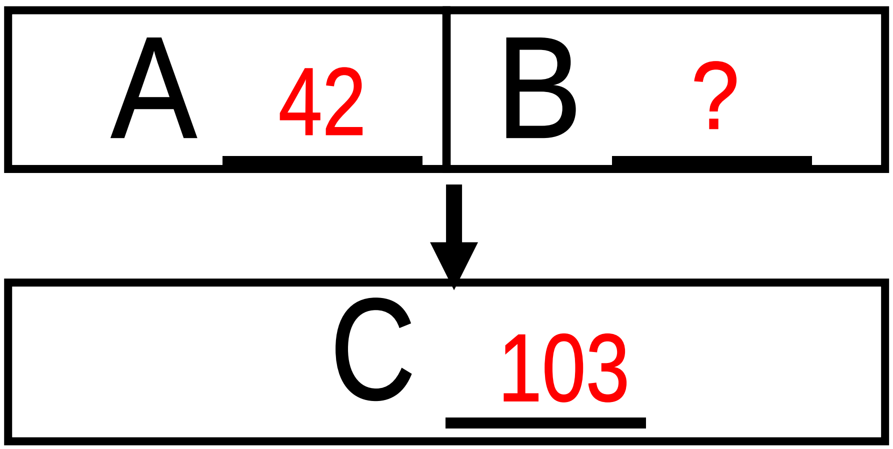
- *Change* Problems: Include increase or decrease of amounts. These problems include a time element
- *Compare* Problems: Involve comparisons of two different types of items in different sets. These problems lack a time element.

Source: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel,B. (2009). Assisting students struggling with mathematics: Response to Intervention Rtl) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/.

Teach Students to Identify 'Underlying Structures' of Word Problems

Change Problems: Include increase or decrease of amounts. These problems include a time element.

Example: Michael gave his friend Franklin 42 marbles to add to his collection. After receiving the new marbles, Franklin had 103 marbles in his collection. How many marbles did Franklin have before Michael's gift?

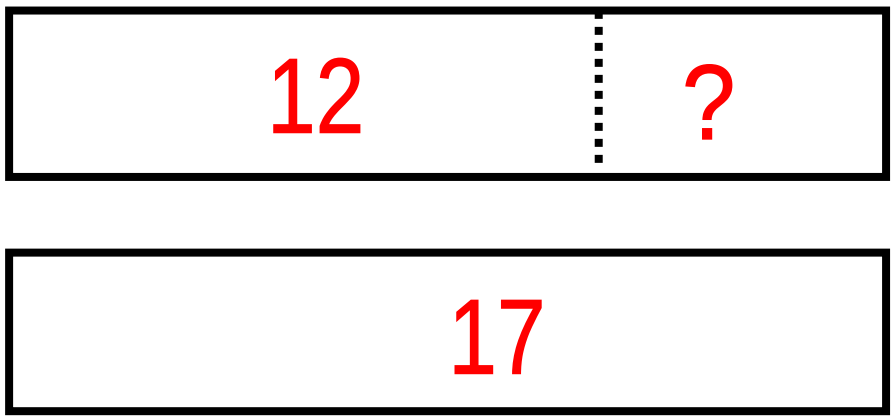


Source: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel,B. (2009). Assisting students struggling with mathematics: Response to Intervention RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/.

Teach Students to Identify 'Underlying Structures' of Word Problems

Compare Problems: Involve comparisons of two different types of items in different sets. These problems lack a time element.

Example: In the zoo, there are 12 antelope and 17 alligators. How many more alligators than antelope are there in the zoo?



Source: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to Intervention Rtl) for elementary and middle schools (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/.

How Do We Reach Low-Performing Math Students?: Instructional Recommendations

Important elements of math instruction for low-performing students:

- "Providing teachers and students with data on student performance"
- "Using peers as tutors or instructional guides"
- "Providing clear, specific feedback to parents on their children's mathematics success"
- "Using principles of explicit instruction in teaching math concepts and procedures." p. 51

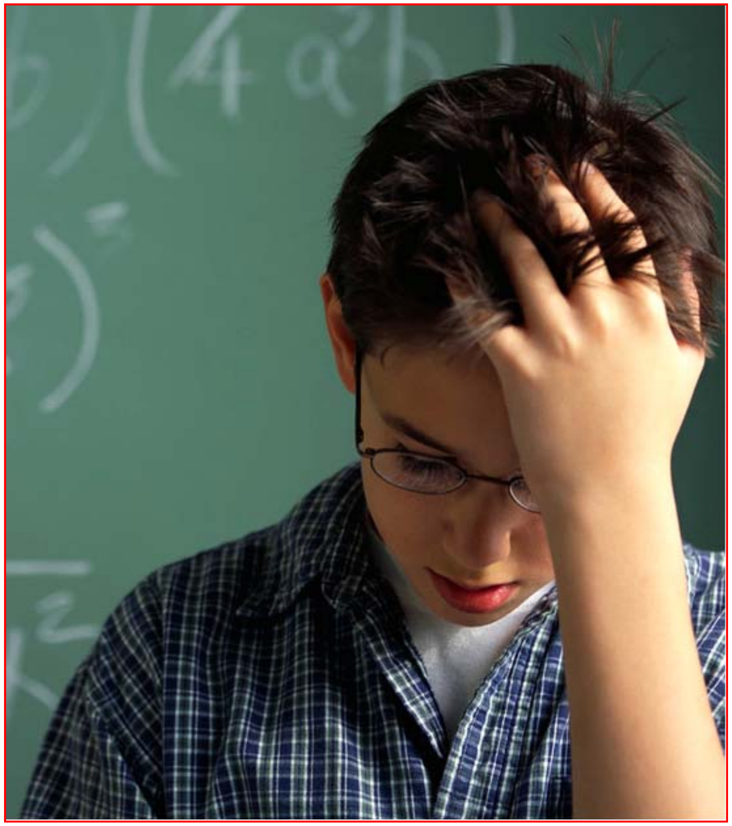
Source: Baker, S., Gersten, R., & Lee, D. (2002).A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal, 103*(1), 51-73..

Activity: How Do We Reach Low-Performing Students?

- Review the handout on p. 10 of your packet and consider each of the elements found to benefit low-performing math students.
- For each element, brainstorm ways that you could promote this idea in your math classroom.

How Do We Reach Low-Performing Math Students?: Instructional Recommendations *Important elements of math instruction for low-performing students* (Baker, Gersten, & Lee, 2002; p. 51):	***IDEAS FOR IMPLEMENTATION***
"Providing teachers and students with data on student performance"	
"Using peers as tutors or instructional guides"	
"Providing clear, specific feedback to parents on their children's mathematics success"	
"Using principles of explicit instruction in teaching math concepts and procedures."	

RTI Challenge: Understanding the Student With 'Math Difficulties'



Who is At Risk for Poor Math Performance?: A Proactive Stance

"…we use the term mathematics difficulties rather than mathematics disabilities. Children who exhibit mathematics difficulties include those performing in the low average range (e.g., at or below the 35th percentile) as well as those performing well below average…Using higher percentile cutoffs increases the likelihood that young children who go on to have serious math problems will be picked up in the screening." p. 295

Source: *Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. Journal of Learning Disabilities, 38, 293-304.*

Three General Levels of Math Skill Development

(Kroesbergen & Van Luit, 2003)

As students move from lower to higher grades, they move through levels of acquisition of math skills, to include:

- Number sense
- Basic math operations (i.e., addition, subtraction, multiplication, division)
- Problem-solving skills: "The solution of both verbal and nonverbal problems through the application of previously acquired information" (Kroesbergen & Van Luit, 2003, p. 98)

Source: Kroesbergen, E., & Van Luit, J. E. H. (2003). Mathematics interventions for children with special educational needs. Remedial and Special Education, 24, 97-114..

What is 'Number Sense'?

(Clarke & Shinn, 2004)

"… the ability to understand the meaning of numbers and define different relationships among numbers.

Children with number sense can recognize the relative size of numbers, use referents for measuring objects and events, and think and work with numbers in a flexible manner that treats numbers as a sensible system." p. 236

Source: Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. School Psychology Review, 33, 234–248.

What Are Stages of 'Number Sense'?

(Berch, 2005, p. 336)

1. **Innate Number Sense.** Children appear to possess 'hard-wired' ability (neurological 'foundation structures') to acquire number sense. Children's innate capabilities appear also to include the ability to 'represent general amounts', not specific quantities. This innate number sense seems to be characterized by skills at estimation ('approximate numerical judgments') and a counting system that can be described loosely as '1, 2, 3, 4, … a lot'.
2. **Acquired Number Sense.** Young students learn through indirect and direct instruction to count specific objects beyond four and to internalize a number line as a mental representation of those precise number values.

Source: Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*, 333-339...

Task Analysis of Number Sense & Operations

(Methe & Riley-Tillman, 2008)

1. Counting
2. **Comparing and Ordering**: Ability to compare relative amounts e.g., more or less than; ordinal numbers: e.g., first, second, third)
3. **Equal partitioning:** Dividing larger set of objects into ‘equal parts’
4. **Composing and decomposing**: Able to create different subgroupings of larger sets (for example, stating that a group of 10 objects can be broken down into 6 objects and 4 objects or 3 objects and 7 objects)
5. **Grouping and place value:** “abstractly grouping objects into sets of 10” (p. 32) in base-10 counting system.
6. **Adding to/taking away**: Ability to add and subtract amounts from sets “by using accurate strategies that do not rely on laborious enumeration, counting, or equal partitioning.” P. 32

Source: *Methe, S. A., & Riley-Tillman, T. C. (2008). An informed approach to selecting and designing early mathematics interventions. School Psychology Forum: Research into Practice, 2, 29-41.*

Children's Understanding of Counting Rules

The development of children's counting ability depends upon the development of:

- **One-to-one correspondence:** "one and only one word tag, e.g., 'one,' 'two,' is assigned to each counted object".
- **Stable order:** "the order of the word tags must be invariant across counted sets".
- **Cardinality:** "the value of the final word tag represents the quantity of items in the counted set".
- **Abstraction:** "objects of any kind can be collected together and counted".
- **Order irrelevance:** "items within a given set can be tagged in any sequence".

Source: Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*, 4-15.

Computation Fluency: Benefits of Automaticity of 'Arithmetic Combinations' (Gersten, Jordan, & Flojo, 2005)

- There is a strong correlation between poor retrieval of arithmetic combinations ('math facts') and global math delays
- Automatic recall of arithmetic combinations frees up student 'cognitive capacity' to allow for understanding of higher-level problem-solving
- By internalizing numbers as mental constructs, students can manipulate those numbers in their head, allowing for the intuitive understanding of arithmetic properties, such as *associative property* and *commutative property*

Source: *Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. Journal of Learning Disabilities, 38, 293-304.*

How much is 3 + 8?: Strategies to Solve…

Least efficient strategy: Count out and group 3 objects; count out and group 8 objects; count all objects:

 +  =11

More efficient strategy: Begin at the number 3 and ‘count up’ 8 more digits (often using fingers for counting): 3 + 8

More efficient strategy: Begin at the number 8 (larger number) and ‘count up’ 3 more digits: 8 + 3

Most efficient strategy: ‘3 + 8’ arithmetic combination is stored in memory and automatically retrieved: Answer = 11

Source: *Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. Journal of Learning Disabilities, 38, 293-304.*

Math Skills: Importance of Fluency in Basic Math Operations

"[A key step in math education is] to learn the four basic mathematical operations (i.e., addition, subtraction, multiplication, and division). Knowledge of these operations and a capacity to perform mental arithmetic play an important role in the development of children's later math skills. Most children with math learning difficulties are unable to master the four basic operations before leaving elementary school and, thus, need special attention to acquire the skills. A … category of interventions is therefore aimed at the acquisition and automatization of basic math skills."

Source: *Kroesbergen, E., & Van Luit, J. E. H. (2003). Mathematics interventions for children with special educational needs. Remedial and Special Education, 24, 97-114.*

Profile of Students With Significant Math Difficulties

Spatial organization. The student commits errors such as misaligning numbers in columns in a multiplication problem or confusing directionality in a subtraction problem (and subtracting the original number—minuend—from the figure to be subtracted (subtrahend).

Visual detail. The student misreads a mathematical sign or leaves out a decimal or dollar sign in the answer.

Procedural errors. The student skips or adds a step in a computation sequence. Or the student misapplies a learned rule from one arithmetic procedure when completing another, different arithmetic procedure.

Inability to 'shift psychological set'. The student does not shift from one operation type (e.g., addition) to another (e.g., multiplication) when warranted.

Graphomotor. The student's poor handwriting can cause him or her to misread handwritten numbers, leading to errors in computation.

Memory. The student fails to remember a specific math fact needed to solve a problem. (The student may KNOW the math fact but not be able to recall it at 'point of performance'.)

Judgment and reasoning. The student comes up with solutions to problems that are clearly unreasonable. However, the student is not able adequately to evaluate those responses to gauge whether they actually make sense in context.

Source: Rourke, B. P. (1993). Arithmetic disabilities, specific & otherwise: A neuropsychological perspective. *Journal of Learning Disabilities*, 26, 214-226.

Activity: Profile of Math Difficulties

- Review the *profile of students with significant math difficulties* that appears on p. 9 of your handout.
- For each item in the profile, discuss what methods you might use to discover whether a particular student experiences this difficulty. Jot your ideas in the 'NOTES' column.

Profile of Students With Significant Math Difficulties (Rourke, 1993)	*NOTES*
Spatial organization. The student commits errors such as misaligning numbers in columns in a multiplication problem or confusing directionality in a subtraction problem (and subtracting the original number—minuend—from the figure to be subtracted (subtrahend).	
Visual detail. The student misreads a mathematical sign or leaves out a decimal or dollar sign in the answer.	
Procedural errors. The student skips or adds a step in a computation sequence. Or the student misapplies a learned rule from one arithmetic procedure when completing another, different arithmetic procedure.	
Inability to 'shift psychological set'. The student does not shift from one operation type (e.g., addition) to another (e.g., multiplication) when warranted.	
Graphomotor. The student's poor handwriting can cause him or her to misread handwritten numbers, leading to errors in computation.	
Memory. The student fails to remember a specific math fact needed to solve a problem. (The student may KNOW the math fact but not be able to recall it at 'point of performance'.)	
Judgment and reasoning. The student comes up with solutions to problems that are clearly unreasonable. However, the student is not able adequately to evaluate those responses to gauge whether they actually make sense in context.	

RTI Challenge: Finding Effective, Research-Based Math Interventions

The Key Role of Classroom Teachers as 'Interventionists' in RTI: 6 Steps

1. The teacher defines the student academic or behavioral problem clearly.
2. The teacher decides on the best explanation for why the problem is occurring.
3. The teacher selects 'evidence-based' interventions.
4. The teacher documents the student's Tier 1 intervention plan.
5. The teacher monitors the student's response (progress) to the intervention plan.
6. The teacher knows what the next steps are when a student fails to make adequate progress with Tier 1 interventions alone.

Big Ideas: The Four Stages of Learning Can Be Summed Up in the 'Instructional Hierarchy'

(Haring et al., 1978)

Student learning can be thought of as a multi-stage process. The universal stages of learning include:

- Acquisition: The student is just acquiring the skill.
- Fluency: The student can perform the skill but must make that skill 'automatic'.
- Generalization: The student must perform the skill across situations or settings.
- Adaptation: The student confronts novel task demands that require that the student adapt a current skill to meet new requirements.

The stage of the learner can determine the appropriate intervention.

Source: Haring, N.G., Lovitt, T.C., Eaton, M.D., & Hansen, C.L. (1978). The fourth R: Research in the classroom. Columbus, OH: Charles E. Merrill Publishing Co.

Math Challenge: The student has not yet *acquired* math facts.

Solution: Use these strategies:

- Incremental Rehearsal
- Cover-Copy-Compare
- Errorless-Learning Worksheets
- Peer Tutoring in Math Computation with Constant Time Delay

Acquisition Stage: Math Review: Incremental Rehearsal of 'Math Facts'

Step 1: The tutor writes down on a series of index cards the math facts that the student needs to learn. The problems are written without the answers.

4 x 5 =__	2 x 6 =__	5 x 5 =__
3 x 2 =__	3 x 8 =__	5 x 3 =__
6 x 5 =__	9 x 2 =__	3 x 6 =__
8 x 2 =__	4 x 7 =__	8 x 4 =__
9 x 7 =__	7 x 6 =__	3 x 5 =__

Math Review: Incremental Rehearsal of 'Math Facts'

Step 2: The tutor reviews the 'math fact' cards with the student. Any card that the student can answer within 2 seconds is sorted into the 'KNOWN' pile. Any card that the student cannot answer within two seconds—or answers incorrectly—is sorted into the 'UNKNOWN' pile.

'KNOWN' Facts		'UNKNOWN' Facts
4 x 5 =__	2 x 6 =__	3 x 8 =__
3 x 2 =__	5 x 3 =__	9 x 2 =__
3 x 6 =__	8 x 4 =__	5 x 5 =__
6 x 5 =__	4 x 7 =__	8 x 2 =__
9 x 7 =__	7 x 6 =__	3 x 5 =__

Math Review: Incremental Rehearsal of ‘Math Facts’

Step 3: The tutor is now ready to follow a nine-step incremental-rehearsal sequence: First, the tutor presents the student with a single index card containing an ‘unknown’ math fact. The tutor reads the problem aloud, gives the answer, then prompts the student to read off the same unknown problem and provide the correct answer.

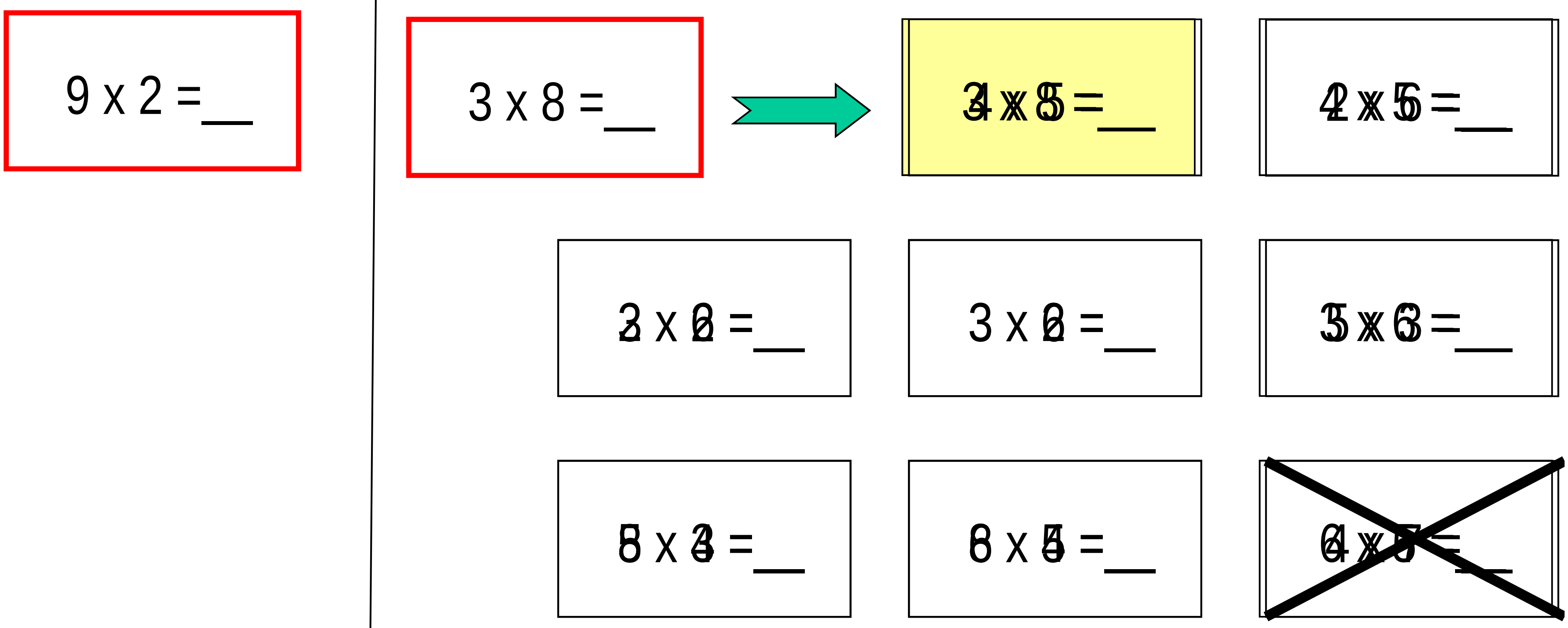
[illegible] review deck contains a total of one ‘unknown’ and nine ‘known’ math facts (a ratio of 90 percent ‘known’ to 10 percent ‘unknown’ material)

3 x 8 =__	4 x 5 =__	2 x 6 =__
3 x 2 =__	3 x 6 =__	5 x 3 =__
8 x 4 =__	6 x 5 =__	4 x 7 =__

Math Review: Incremental Rehearsal of 'Math Facts'

Step 4: At this point, the last 'known' math fact that had been added to the student's review deck is discarded (placed back into the original pile of 'known' problems) and the previously 'unknown' math fact is now treated as the first 'known' math fact in new student review deck for future drills.

Daily review sessions are discontinued either when the ... out or when the student answers an 'unknown' math fact incorrectly three times.



Acquisition Stage: Cover-Copy-Compare:

Math Computational Fluency-Building Intervention

The student is given sheet with correctly completed math problems in left column and index card.

For each problem, the student:

- studies the model
- covers the model with index card
- copies the problem from memory
- solves the problem
- uncovers the correctly completed model to check answer

Source: *Skinner, C.H., Turco, T.L., Beatty, K.L., & Rasavage, C. (1989). Cover, copy, and compare: A method for increasing multiplication performance. School Psychology Review, 18, 412-420.*

Acquisition Stage: Math Computation: Motivate With 'Errorless Learning' Worksheets

In this version of an 'errorless learning' approach, the student is directed to complete math facts as quickly as possible. If the student comes to a number problem that he or she cannot solve, the student is encouraged to locate the problem and its correct answer in the key at the top of the page and write it in. This idea works best for basic math facts.

Such speed drills build computational fluency while promoting students' ability to visualize and to use a mental number line.

TIP: Consider turning this activity into a 'speed drill'. The student is given a kitchen timer and instructed to set the timer for a predetermined span of time (e.g., 2 minutes) for each drill. The student completes as many problems as possible before the timer rings. The student then graphs the number of problems correctly computed each day on a time-series graph, attempting to better his or her previous score.

Source: *Caron, T. A. (2007). Learning multiplication the easy way. The Clearing House, 80, 278-282*

'Errorless Learning' Worksheet Sample

Curriculum-Based Assessment Mathematics
Multiple-Skills Computation Probe: Examiner Copy

Item 1: 2 CD/2 CD Total *SUBTRACTION: 1-digit number from a 1- or 2-digit number: no regrouping*	Item 2: 2 CD/4 CD Total *MULTIPLICATION: 2-digit number times 1-digit number: no regrouping*	Item 3: 2 CD/6 CD Total *SUBTRACTION: 1-digit number from a 1- or 2-digit number: no regrouping*
$\begin{array}{r} 26 \\ -\ 11 \\ \hline \mathbf{15} \end{array}$	$\begin{array}{r} 34 \\ \times\ 2 \\ \hline \mathbf{68} \end{array}$	$\begin{array}{r} 38 \\ -\ 22 \\ \hline \mathbf{16} \end{array}$
$\begin{array}{r} 26 \\ -11 \\ \hline \end{array}$	$\begin{array}{r} 34 \\ \times\ 2 \\ \hline \end{array}$	$\begin{array}{r} 38 \\ -22 \\ \hline \end{array}$

Source: *Caron, T. A. (2007). Learning multiplication the easy way. The Clearing House, 80, 278-282*

Peer Tutoring in Math Computation with Constant Time Delay

Peer Tutoring in Math Computation with Constant Time Delay

- DESCRIPTION: This intervention employs students as reciprocal peer tutors to target acquisition of basic math facts (math computation) using constant time delay (Menesses & Gresham, 2009; Telecsan, Slaton, & Stevens, 1999). Each tutoring 'session' is brief and includes its own progress-monitoring component--making this a convenient and time-efficient math intervention for busy classrooms.

Peer Tutoring in Math Computation with Constant Time Delay

MATERIALS:

Student Packet: A work folder is created for each tutor pair. The folder contains:

- ❑ 10 math fact cards with equations written on the front and correct answer appearing on the back. NOTE: The set of cards is replenished and updated regularly as tutoring pairs master their math facts.
- ❑ Progress-monitoring form for each student.
- ❑ Pencils.

Peer Tutoring in Math Computation with Constant Time Delay

PREPARATION: To prepare for the tutoring program, the teacher selects students to participate and trains them to serve as tutors.

Select Student Participants. Students being considered for the reciprocal peer tutor program should at minimum meet these criteria (Telecsan, Slaton, & Stevens, 1999, Menesses & Gresham, 2009):

- ❑ Is able and willing to follow directions;
- ❑ Shows generally appropriate classroom behavior;
- ❑ Can attend to a lesson or learning activity for at least 20 minutes.

Peer Tutoring in Math Computation with Constant Time Delay

Select Student Participants (Cont.). Students being considered for the reciprocal peer tutor program should at minimum meet these criteria (Telecsan, Slaton, & Stevens, 1999, Menesses & Gresham, 2009):

- Is able to name all numbers from 0 to 18 (if tutoring in addition or subtraction math facts) and name all numbers from 0 to 81 (if tutoring in multiplication or division math facts).
- Can correctly read aloud a sampling of 10 math-facts (equation plus answer) that will be used in the tutoring sessions. (NOTE: The student does not need to have memorized or otherwise mastered these math facts to participate—just be able to read them aloud from cards without errors).
- [To document a deficit in math computation] When given a two-minute math computation probe to complete independently, computes fewer than 20 correct digits (Grades 1-3) or fewer than 40 correct digits (Grades 4 and up) (Deno & Mirkin, 1977).

Peer Tutoring in Math Computation: Teacher Nomination Form

Reciprocal Peer Tutoring in Math Computation: Teacher Nomination Form

Teacher: ______________________ Classroom: ______________ Date: ______________

Directions: Select students in your class that you believe would benefit from participation in a peer tutoring program to boost math computation skills. Write the names of your student nominees in the space provided below. Remember, students who are considered for the peer tutoring program should—*at minimum*—meet these criteria:

- Show generally appropriate classroom behaviors and follow directions.
- Can pay attention to a lesson or learning activity for at least 20 minutes.
- Are able to wait appropriately to hear the correct answer from the tutor if the student does not know the answer.
- When given a two-minute math computation probe to complete independently, computes **fewer** than 20 correct digits (Grades 1-3) or **fewer** than 40 correct digits (Grades 4 and up) (Deno & Mirkin, 1977).
- Can name all numbers from 0 to 18 (if tutoring in addition or subtraction math facts) and name all numbers from 0 to 81 (if tutoring in multiplication or division math facts).
- Can correctly read aloud a sampling of 10 math-facts (equation plus answer) that will be used in the tutoring sessions. (NOTE: The student does not need to have memorized or otherwise mastered these math facts to participate—just be able to read them aloud from cards without errors).

Number	Student Name	NOTES
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		

Peer Tutoring in Math Computation with Constant Time Delay

Tutoring Activity. Each tutoring 'session' last for 3 minutes. The tutor:

- *Presents Cards*. The tutor presents each card to the tutee for 3 seconds.
- *Provides Tutor Feedback*. [When the tutee responds correctly] The tutor acknowledges the correct answer and presents the next card.

  [When the tutee does not respond within 3 seconds or responds incorrectly] The tutor states the correct answer and has the tutee repeat the correct answer. The tutor then presents the next card.
- *Provides Praise*. The tutor praises the tutee immediately following correct answers.
- *Shuffles Cards*. When the tutor and tutee have reviewed all of the math-fact carts, the tutor shuffles them before again presenting cards.

Peer Tutoring in Math Computation with Constant Time Delay

Progress-Monitoring Activity. The tutor concludes each 3-minute tutoring session by assessing the number of math facts mastered by the tutee. The tutor follows this sequence:

- *Presents Cards.* The tutor presents each card to the tutee for 3 seconds.
- *Remains Silent*. The tutor does not provide performance feedback or praise to the tutee, or otherwise talk during the assessment phase.
- *Sorts Cards*. Based on the tutee's responses, the tutor sorts the math-fact cards into 'correct' and 'incorrect' piles.
- *Counts Cards and Records Totals*. The tutor counts the number of cards in the 'correct' and 'incorrect' piles and records the totals on the tutee's progress-monitoring chart.

Peer Tutoring in Math Computation with Constant Time Delay

Tutoring Integrity Checks. As the student pairs complete the tutoring activities, the supervising adult monitors the integrity with which the intervention is carried out. At the conclusion of the tutoring session, the adult gives feedback to the student pairs, praising successful implementation and providing corrective feedback to students as needed. NOTE: Teachers can use the attached form *Peer Tutoring in Math Computation with Constant Time Delay: Integrity Checklist* to conduct integrity checks of the intervention and student progress-monitoring components of the math peer tutoring.

Peer Tutoring in Math Computation: Intervention Integrity Sheet: (Part 1: Tutoring Activity)

Peer Tutoring in Math Computation with Constant Time Delay: Integrity Checklist			
Tutoring Session: Intervention Phase			
Directions: Observe the tutor and tutee for a full intervention session. Use this checklist to record whether each of the key steps of the intervention were correctly followed.			
Correctly Carried Out?	Step	Tutor Action	NOTES
___Y___N	1.	**Promptly Initiates Session.** At the start of the timer, the tutor immediately presents the first math-fact card.	
___Y___N	2.	**Presents Cards.** The tutor presents each card to the tutee for 3 seconds.	
___Y___N	3.	**Provides Tutor Feedback.** [When the tutee responds correctly] The tutor acknowledges the correct answer and presents the next card. [When the tutee does not respond within 3 seconds or responds incorrectly] The tutor states the correct answer and has the tutee repeat the correct answer. The tutor then presents the next card.	
___Y___N	4.	**Provides Praise.** The tutor praises the tutee immediately following correct answers.	
___Y___N	5.	**Shuffles Cards.** When the tutor and tutee have reviewed all of the math-fact carts, the tutor shuffles them before again presenting cards.	
___Y___N	6.	**Continues to the Timer.** The tutor continues to presents math-fact cards for tutee response until the timer rings.	

Peer Tutoring in Math Computation: Intervention Integrity Sheet (Part 2: Progress-Monitoring)

Tutoring Session: Assessment Phase			
Directions: Observe the tutor and tutee during the progress-monitoring phase of the session. Use this checklist to record whether each of the key steps of the assessment were correctly followed.			
Correctly Carried Out?	Step	Tutor Action	NOTES
___Y___N	1.	**Presents Cards.** The tutor presents each card to the tutee for 3 seconds.	
___Y___N	2.	**Remains Silent.** The tutor does not provide performance feedback or praise to the tutee, or otherwise talk during the assessment phase.	
___Y___N	3.	**Sorts Cards.** The tutor sorts cards into 'correct' and 'incorrect' piles based on the tutee's responses.	
___Y___N	4.	**Counts Cards and Records Totals.** The tutor counts the number of cards in the 'correct' and 'incorrect' piles and records the totals on the tutee's progress-monitoring chart.	

Peer Tutoring in Math Computation: Score Sheet

Math Tutoring: Score Sheet

Tutor 'Coach': ______________________ Tutee 'Player': ______________________

Directions to the Tutor: Write down the number of math-fact cards that your partner answered *correctly* and the number answered *incorrectly*.		
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:

Team Activity: Peer Tutoring in Math Computation with Constant Time Delay

Groups: At your table:

- Discuss how you might use or adapt this math computation tutoring intervention in your classroom or school.

Math Challenge: The student has acquired math computation skills but is not yet fluent.

Solution: Use these strategies:

- Explicit Time Drills
- Self-Administered Arithmetic Combination Drills With Performance Self-Monitoring & Incentives

Explicit Time Drills:

Math Computational Fluency-Building Intervention

Explicit time-drills are a method to boost students' rate of responding on math-fact worksheets.

The teacher hands out the worksheet. Students are told that they will have 3 minutes to work on problems on the sheet. The teacher starts the stop watch and tells the students to start work. At the end of the first minute in the 3-minute span, the teacher 'calls time', stops the stopwatch, and tells the students to underline the last number written and to put their pencils in the air. Then students are told to resume work and the teacher restarts the stopwatch. This process is repeated at the end of minutes 2 and 3. At the conclusion of the 3 minutes, the teacher collects the student worksheets.

Source: *Rhymer, K. N., Skinner, C. H., Jackson, S., McNeill, S., Smith, T., & Jackson, B. (2002). The 1-minute explicit timing intervention: The influence of mathematics problem difficulty. Journal of Instructional Psychology, 29(4), 305-311.*

Fluency Stage: Math Computation

Self-Administered Arithmetic Combination Drills With Performance Self-Monitoring & Incentives

1. The student is given a math computation worksheet of a specific problem type, along with an answer key [Academic Opportunity to Respond].
2. The student consults his or her performance chart and notes previous performance. The student is encouraged to try to 'beat' his or her most recent score.
3. The student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems as possible. The student sets a timer and works on the computation sheet until the timer rings. [Active Student Responding]
4. The student checks his or her work, giving credit for each ***correct digit*** (digit of correct value appearing in the correct place-position in the answer). [Performance Feedback]
5. The student records the day's score of TOTAL number of correct digits on his or her personal performance chart.
6. The student receives praise or a reward if he or she exceeds the most recently posted number of correct digits.

Application of 'Learn Unit' framework from : *Heward, W.L. (1996). Three low-tech strategies for increasing the frequency of active student response during group instruction. In R. Gardner, D. M.S ainato, J. O. Cooper, T. E. Heron, W. L. Heward, J. W. Eshleman,& T. A. Grossi (Eds.), Behavior analysis in education: Focus on measurably superior instruction (pp.283-320). Pacific Grove, CA:Brooks/Cole.*

Self-Administered Arithmetic Combination Drills: Examples of Student Worksheet and Answer Key

Curriculum-Based Assessment Mathematics
Single-Skill Computation Probe: Student Copy

Student: Date: ________________

8 x6	3 x8	2 x3
9 x5	6 x5	1 x3
3 x8	3 x7	2 x4

www.interventioncentral.org

Curriculum-Based Assessment Mathematics
Single-Skill Computation Probe: Examiner Copy

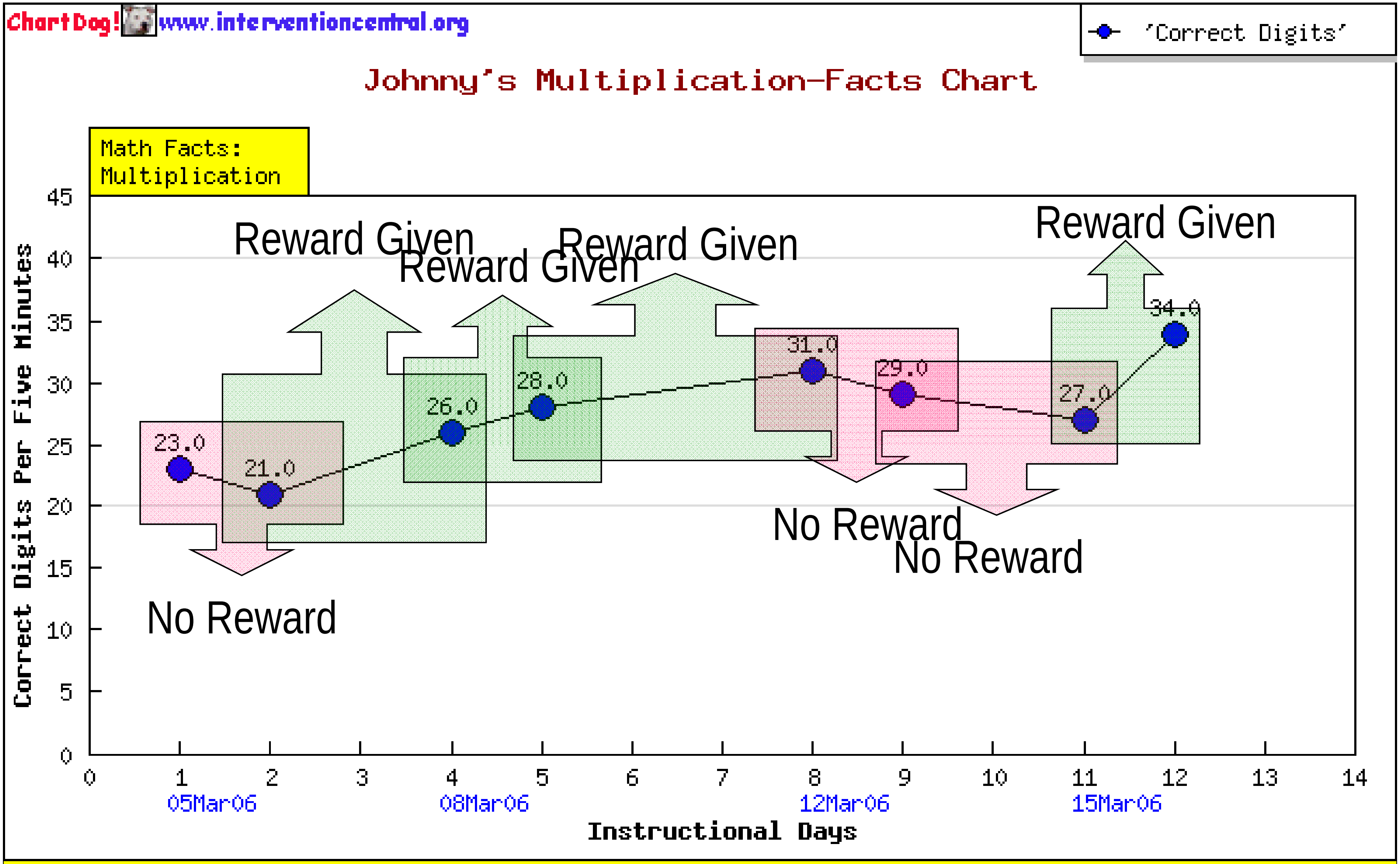
MULTIPLICATION: Multiplication facts: 0 to 9

Item 1: 2 CD/2 CD total 8 x 6 48	Item 2: 2 CD/4 CD total 3 x 8 24	Item 3: 1 CD/5 CD total 2 x 3 6
Item 4: 2 CD/7 CD total 9 x 5 45	Item 5: 2 CD/9 CD total 6 x 5 30	Item 6: 1 CD/10 CD total 1 x 3 3
Item 7: 2 CD/12 CD total 3 x 8 24	Item 8: 2 CD/14 CD total 3 x 7 21	Item 9: 1 CD/15 CD total 2 x 4 8

www.interventioncentral.org
Click for Student Worksheet

Worksheets created using Math Worksheet Generator. Available online at:
http://www.interventioncentral.org/htmdocs/tools/mathprobe/addsing.php

Self-Administered Arithmetic Combination Drills…



Math Challenge: The student is not motivated to attempt math facts.

Solution: Use these strategies:

- Chunking
- Problem-Interspersal Technique

Motivation: Math Computation: Chunking

- Break longer assignments into shorter assignments with performance feedback given after each shorter ‘chunk’ (e.g., break a 20-minute math computation worksheet task into 3 seven-minute assignments). Breaking longer assignments into briefer segments also allows the teacher to praise struggling students more frequently for work completion and effort, providing an additional ‘natural’ reinforcer.

Source: *Skinner, C. H., Pappas, D. N., & Davis, K. A. (2005). Enhancing academic engagement: Providing opportunities for responding and influencing students to choose to respond. Psychology in the Schools, 42, 389-403.*

Motivation: Math Computation: Problem Interspersal Technique

- The teacher first identifies the range of ‘challenging’ problem-types (number problems appropriately matched to the student’s current instructional level) that are to appear on the worksheet.
- Then the teacher creates a series of ‘easy’ problems that the students can complete very quickly (e.g., adding or subtracting two 1-digit numbers). The teacher next prepares a series of student math computation worksheets with ‘easy’ computation problems interspersed at a fixed rate among the ‘challenging’ problems.
- If the student is expected to complete the worksheet independently, ‘challenging’ and ‘easy’ problems should be interspersed at a 1:1 ratio (that is, every ‘challenging’ problem in the worksheet is preceded and/or followed by an ‘easy’ problem).
- If the student is to have the problems read aloud and then asked to solve the problems mentally and write down only the answer, the items should appear on the worksheet at a ratio of 3 ‘challenging’ problems for every ‘easy’ one (that is, every 3 ‘challenging’ problems are preceded and/or followed by an ‘easy’ one).

Source: *Hawkins, J., Skinner, C. H., & Oliver, R. (2005). The effects of task demands and additive interspersal ratios on fifth-grade students’ mathematics accuracy. School Psychology Review, 34, 543-555..*

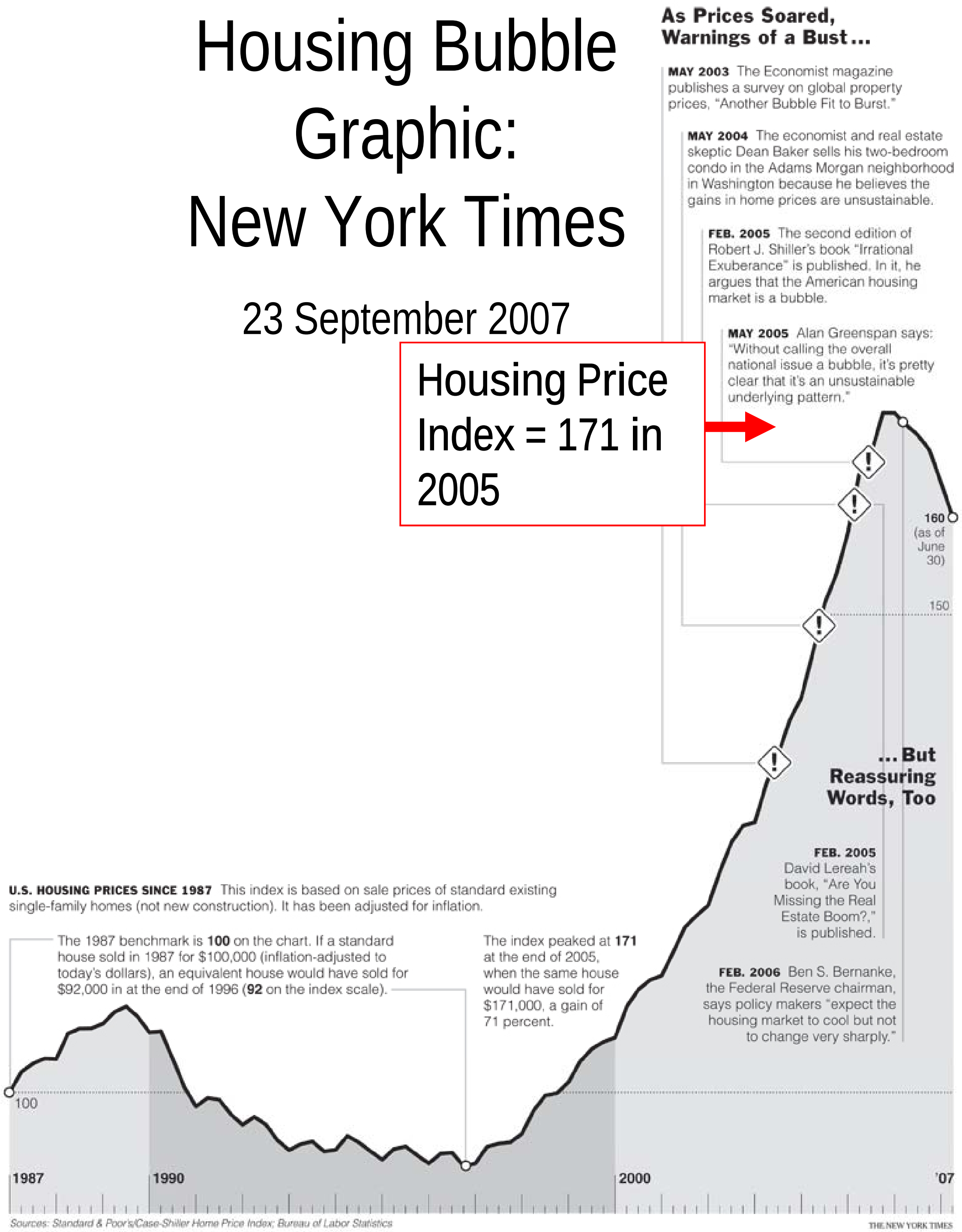
Math Challenge: The student misinterprets math graphics.

Solution:

- Use Question-Answer Relationships (QARs) to interpret information from math graphics

Housing Bubble Graphic: New York Times

23 September 2007



Housing Price Index = 171 in 2005

Housing Price Index = 100 in 1987

Classroom Challenges in Interpreting Math Graphics

When encountering math graphics, students may :

- expect the answer to be easily accessible when in fact the graphic may expect the reader to interpret and draw conclusions
- be inattentive to details of the graphic
- treat irrelevant data as ‘relevant’
- not pay close attention to questions before turning to graphics to find the answer
- fail to use their prior knowledge both to extend the information on the graphic and to act as a possible ‘check’ on the information that it presents.

Source: *Mesmer, H.A.E., & Hutchins, E.J. (2002). Using QARs with charts and graphs. The Reading Teacher, 56, 21–27.*

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics

Students can be more savvy interpreters of graphics in applied math problems by applying the Question-Answer Relationship (QAR) strategy. Four Kinds of QAR Questions:

- RIGHT THERE questions are fact-based and can be found in a single sentence, often accompanied by 'clue' words that also appear in the question.
- THINK AND SEARCH questions can be answered by information in the text but require the scanning of text and making connections between different pieces of factual information.
- AUTHOR AND YOU questions require that students take information or opinions that appear in the text and combine them with the reader's own experiences or opinions to formulate an answer.
- ON MY OWN questions are based on the students' own experiences and do not require knowledge of the text to answer.

Source: *Mesmer, H.A.E., & Hutchins, E.J. (2002). Using QARs with charts and graphs. The Reading Teacher, 56, 21–27.*

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

1. DISTINGUISHING DIFFERENT KINDS OF GRAPHICS. Students are taught to differentiate between common types of graphics: e.g., table (grid with information contained in cells), chart (boxes with possible connecting lines or arrows), picture (figure with labels), line graph, bar graph.

   Students note significant differences between the various graphics, while the teacher records those observations on a wall chart. Next students are given examples of graphics and asked to identify which general kind of graphic each is.

   Finally, students are assigned to go on a 'graphics hunt', locating graphics in magazines and newspapers, labeling them, and bringing to class to review.

Source: *Mesmer, H.A.E., & Hutchins, E.J. (2002). Using QARs with charts and graphs. The Reading Teacher, 56, 21–27.*

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

2. INTERPRETING INFORMATION IN GRAPHICS. Students are paired off, with stronger students matched with less strong ones. The teacher spends at least one session presenting students with examples from each of the graphics categories.

   The presentation sequence is ordered so that students begin with examples of the most concrete graphics and move toward the more abstract: Pictures > tables > bar graphs > charts > line graphs.

   At each session, student pairs examine graphics and discuss questions such as: "What information does this graphic present? What are strengths of this graphic for presenting data? What are possible weaknesses?"

Source: *Mesmer, H.A.E., & Hutchins, E.J. (2002). Using QARs with charts and graphs. The Reading Teacher, 56, 21–27.*

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

3. LINKING THE USE OF QARS TO GRAPHICS. Students are given a series of data questions and correct answers, with each question accompanied by a graphic that contains information needed to formulate the answer.

   Students are also each given index cards with titles and descriptions of each of the 4 QAR questions: RIGHT THERE, THINK AND SEARCH, AUTHOR AND YOU, ON MY OWN.

   Working in small groups and then individually, students read the questions, study the matching graphics, and ‘verify’ the answers as correct. They then identify the type question being asked using their QAR index cards.

Source: *Mesmer, H.A.E., & Hutchins, E.J. (2002). Using QARs with charts and graphs. The Reading Teacher, 56, 21–27.*

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

4. USING QARS WITH GRAPHICS INDEPENDENTLY. When students are ready to use the QAR strategy independently to read graphics, they are given a laminated card as a reference with 6 steps to follow:

   A. *Read the question,*
   B. *Review the graphic,*
   C. *Reread the question,*
   D. *Choose a QAR,*
   E. *Answer the question, and*
   F. *Locate the answer derived from the graphic in the answer choices offered.*

   Students are strongly encouraged NOT to read the answer choices offered until they have first derived their own answer, so that those choices don't short-circuit their inquiry.

Source: *Mesmer, H.A.E., & Hutchins, E.J. (2002). Using QARs with charts and graphs. The Reading Teacher, 56, 21–27.*

Math Challenge: The student fails to use a structured approach to solving word problems.

Solution: Train the student to use a cognitive strategy to attack word problems and to use ‘self-coaching’ (metacognitive techniques) to monitor the problem-solving process.

Importance of Metacognitive Strategy Use…

"Metacognitive processes focus on self-awareness of cognitive knowledge that is presumed to be necessary for effective problem solving, and they direct and regulate cognitive processes and strategies during problem solving…That is, successful problem solvers, consciously or unconsciously (depending on task demands), use self-instruction, self-questioning, and self-monitoring to gain access to strategic knowledge, guide execution of strategies, and regulate use of strategies and problem-solving performance." p. 231

Source: *Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities.* Journal of Learning Disabilities, 25*, 230-248.*

Elements of Metacognitive Processes

"**Self-instruction** helps students to identify and direct the problem-solving strategies prior to execution. **Self-questioning** promotes internal dialogue for systematically analyzing problem information and regulating execution of cognitive strategies. **Self-monitoring** promotes appropriate use of specific strategies and encourages students to monitor general performance. [Emphasis added]." p. 231

Source: *Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities.* Journal of Learning Disabilities, 25*, 230-248.*

Combining Cognitive & Metacognitive Strategies to Assist Students With Mathematical Problem Solving

Solving an advanced math problem independently requires the coordination of a number of complex skills. The following strategies combine both cognitive and metacognitive elements (Montague, 1992; Montague & Dietz, 2009). First, the student is taught a 7-step process for attacking a math word problem (cognitive strategy). Second, the instructor trains the student to use a three-part self-coaching routine for each of the seven problem-solving steps (metacognitive strategy).

Cognitive Portion of Combined Problem Solving Approach

In the cognitive part of this multi-strategy intervention, the student learns an explicit series of steps to analyze and solve a math problem. Those steps include:

1. **Reading the problem.** The student reads the problem carefully, noting and attempting to clear up any areas of uncertainly or confusion (e.g., unknown vocabulary terms).
2. **Paraphrasing the problem.** The student restates the problem in his or her own words.
3. **'Drawing' the problem.** The student creates a drawing of the problem, creating a visual representation of the word problem.
4. **Creating a plan to solve the problem.** The student decides on the best way to solve the problem and develops a plan to do so.
5. **Predicting/Estimating the answer.** The student estimates or predicts what the answer to the problem will be. The student may compute a quick approximation of the answer, using rounding or other shortcuts.
6. **Computing the answer.** The student follows the plan developed earlier to compute the answer to the problem.
7. **Checking the answer.** The student methodically checks the calculations for each step of the problem. The student also compares the actual answer to the estimated answer calculated in a previous step to ensure that there is general agreement between the two values.

Metacognitive Portion of Combined Problem Solving Approach

The metacognitive component of the intervention is a three-part routine that follows a sequence of 'Say', 'Ask, 'Check'. For each of the 7 problem-solving steps reviewed above:

- The student first self-instructs by stating, or 'saying', the purpose of the step ('Say').
- The student next self-questions by 'asking' what he or she intends to do to complete the step ('Ask').
- The student concludes the step by self-monitoring, or 'checking', the successful completion of the step ('Check').

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
1. **Read the problem.**	**'Say' (Self-Instruction) Target:** *The student reads and studies the problem carefully before proceeding.* ***'Ask' (Self-Question) Target:*** *Does the student fully understand the problem?* ***'Check' (Self-Monitor) Target:*** *Proceed only if the problem is understood.*	**Say:** "I will read the problem. I will reread the problem if I don't understand it." **Ask: "**Now that I have read the problem, do I fully understand it?" **Check:** "I understand the problem and will move forward."

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
2. **Paraphrase the problem.**	**'Say' (Self-Instruction) Target:** *The student restates the problem in order to demonstrate understanding.* ***'Ask' (Self-Question) Target:*** *Is the student able to paraphrase the problem?* ***'Check' (Self-Monitor) Target:*** *Ensure that any highlighted key words are relevant to the question.*	**Say:** "I will highlight key words and phrases that relate to the problem question." "I will restate the problem in my own words." **Ask: "**Did I highlight the most important words or phrases in the problem?" **Check:** "I found the key words or phrases that will help to solve the problem."

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
3. **'Draw' the problem.**	**'Say' (Self-Instruction) Target:** *The student creates a drawing of the problem to consolidate understanding.* ***'Ask' (Self-Question) Target:*** *Is there a match between the drawing and the problem?* ***'Check' (Self-Monitor) Target:*** *The drawing includes in visual form the key elements of the math problem.*	**Say:** "I will draw a diagram of the problem." **Ask: "**Does my drawing represent the problem?" **Check:** "The drawing contains the essential parts of the problem."

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
4. **Create a plan to solve the problem.**	**'Say' (Self-Instruction) Target:** *The student generates a plan to solve the problem.* ***'Ask' (Self-Question) Target:*** *What plan will help the student to solve this problem?* ***'Check' (Self-Monitor) Target:*** *The plan is appropriate to solve the problem.*	**Say:** "I will make a plan to solve the problem." **Ask:** "What is the first step of this plan? What is the next step of the plan?" **Check:** "My plan has the right steps to solve the problem."

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
5. **Predict/ estimate the Answer.**	**'Say' (Self-Instruction) Target:** *The student uses estimation or other strategies to predict or estimate the answer.* ***'Ask' (Self-Question) Target:*** *What estimating technique will the student use to predict the answer?* ***'Check' (Self-Monitor) Target:*** *The predicted/estimated answer used all of the essential problem information.*	**Say:** "I will estimate what the answer will be." **Ask:** "What numbers in the problem should be used in my estimation?" **Check:** "I did not skip any important information in my estimation."

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
6. **Compute the answer.**	**'Say' (Self-Instruction) Target:** *The student follows the plan to compute the solution to the problem.* ***'Ask' (Self-Question) Target:*** *Does the answer agree with the estimate?* ***'Check' (Self-Monitor) Target:*** *The steps in the plan were followed and the operations completed in the correct order.*	**Say:** "I will compute the answer to the problem." **Ask:** "Does my answer sound right?" "Is my answer close to my estimate?" **Check:** "I carried out all of the operations in the correct order to solve this problem."

Combined Cognitive & Metacognitive Elements of Strategy

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
7. **Check the answer.**	**'Say' (Self-Instruction) Target:** *The student reviews the computation steps to verify the answer.* ***'Ask' (Self-Question) Target:*** *Did the student check all the steps in solving the problem and are all computations correct?* ***'Check' (Self-Monitor) Target:*** *The problem solution appears to have been done correctly.*	**Say:** "I will check the steps of my answer." **Ask:** "Did I go through each step in my answer and check my work?" **Check:** ""

Applied Problems: Pop Quiz

7-Step Problem-Solving:Process

1. Reading the problem.
2. Paraphrasing the problem.
3. 'Drawing' the problem.
4. Creating a plan to solve the problem.
5. Predicting/Estimating the answer.
6. Computing the answer.
7. Checking the answer.

Directions: As a team, read the following problem. At your tables, apply the 7-step problem-solving (cognitive) strategy to complete the problem. As you complete each step of the problem, apply the 'Say-Ask-Check' metacognitive sequence. Try to complete the entire 7 steps within the time allocated for this exercise.

Q: "To move their armies, the Romans built over 50,000 miles of roads. Imagine driving all those miles! Now imagine driving those miles in the first gasoline-driven car that has only three wheels and could reach a top speed of about 10 miles per hour. For safety's sake, let's bring along a spare tire. As you drive the 50,000 miles, you rotate the spare with the other tires so that all four tires get the same amount of wear. Can you figure out how many miles of wear each tire accumulates?"

A: "Since the four wheels of the three-wheeled car share the journey equally, simply take three-fourths of the total distance (50,000 miles) and you'll get 37,500 miles for each tire."

Source: The Math Forum @ Drexel: Critical Thinking Puzzles/Spare My Brain. Retrieved from http://mathforum.org/k12/k12puzzles/critical.thinking/puzz2.html

RTI Challenge : Screening and Progress-Monitoring for Students With Math Difficulties

Inference: Moving Beyond the Margins of the 'Known'

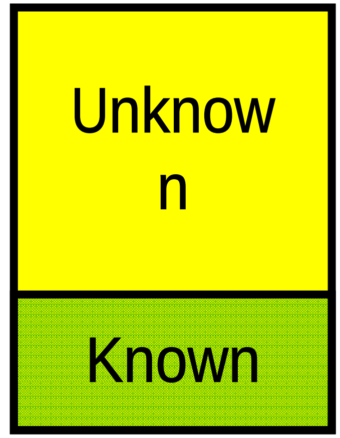
"An inference is a tentative conclusion without direct or conclusive support from available data. All hypotheses are, by definition, inferences. It is critical that problem analysts make distinctions between what is known and what is inferred or hypothesized….Low-level inferences should be exhausted prior to the use of high-level inferences." p. 161

Source: Christ, T. (2008). Best practices in problem analysis. In A. Thomas & J. Grimes (Eds.), Best practices in school psychology V (pp. 159-176).

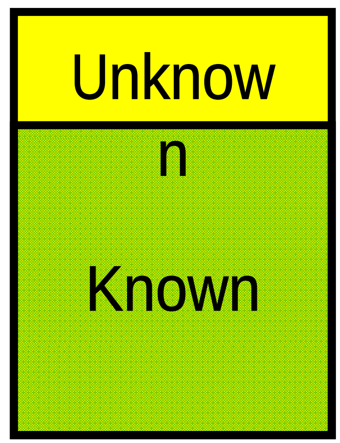
Examples of High vs. Low Inference Hypotheses

The results of grade-wide benchmarking in math computation show that a target 2nd-grade student computes math facts (‘double-digit subtraction without regrouping’) at approximately half the rate of the median child in the grade.

High-Inference Hypothesis. The student has visual processing and memory issues that prevent him or her from proficiently solving math facts. The student requires a multisensory approach such as TouchMath to master the math facts.



Low-Inference Hypothesis. The student has acquired the basic academic skill but needs to build fluency. The student will benefit from repeated opportunities to practice the skill with performance feedback about both accuracy and fluency (e.g., Explicit Time Drill). .



Educational Decisions and Corresponding Types of Assessment

- SCREENING/BENCHMARKING DECISIONS: Tier 1: Brief screenings to quickly indicate whether students in the general-education population are academically proficient or at risk.
- PROGRESS-MONITORING DECISIONS: At Tiers 1, 2, and 3, ongoing 'formative' assessments to judge whether students on intervention are making adequate progress.
- INSTRUCTIONAL/DIAGNOSTIC DECISIONS: At any Tier, detailed assessment to map out specific academic deficits , discover the root cause(s) of a student's academic problem.
- OUTCOME DECISIONS: Summative assessment (e.g., state tests) to evaluate the effectiveness of a program.

Source: Hosp, M. K., Hosp, J. L., & Howell, K. W. (2007). The ABCs of CBM: A practical guide to curriculum-based measurement. New York: Guilford Press.

Clearinghouse for RTI Screening and Progress-Monitoring Tools

- The National Center on RTI (www.rti4success.org) maintains pages rating the technical adequacy of RTI screening and

Tools	Area	Reliability of the Performance Level Score	Reliability of the Slope	Validity of the Performance Level Score	Predictive Validity of the Slope of Improvement	Alternate Forms	Sensitive to Student Improvement	End-of-Year Bench-marks	Rates of Improvement Specified	Norms Disaggregated for Diverse Populations	Disaggregated Reliability and Validity Data	COMPARE RESET
AIMSweb	Math	●	●	●	●	◐	◐	●	●	No	●	☐
AIMSweb	Oral Reading	●	●	●	●	●	◐	●	●	No	●	☐
AIMSweb	Test of Early Literacy - Letter Naming Fluency	●	●	●	●	●	◐	●	●	No	●	☐
AIMSweb	Test of Early Literacy - Letter Sound Fluency	●	●	●	●	●	◐	●	●	No	●	☐

Curriculum-Based Measurement: Advantages as a Set of Tools to Monitor RTI/Academic Cases

- **Aligns** with curriculum-goals and materials
- Is **reliable** and **valid** (has 'technical adequacy')
- Is **criterion-referenced**: sets specific performance levels for specific tasks
- Uses **standard procedures** to prepare materials, administer, and score
- Samples student performance to give objective, observable **'low-inference' information** about student performance
- Has **decision rules** to help educators to interpret student data and make appropriate instructional decisions
- Is **efficient** to implement in schools (e.g., training can be done quickly; the measures are brief and feasible for classrooms, etc.)
- Provides data that can be converted into **visual displays** for ease of communication

Source: Hosp, M.K., Hosp, J. L., & Howell, K. W. (2007). *The ABCs of CBM*. New York: Guilford.

Collective Formative Math Assessment Data to Inform Instruction/Intervention

- Formative math measures have been developed to measure various aspects of math proficiency:
  - CBM: Early Math Fluency (Number Sense)
  - CBM: Math Computation Fluency: Math Facts
  - CBM: Math Concepts & Applications
  - CBM: Math Vocabulary
  - Checklist: Cognitive Strategies

numberfly

The application to create CBM Early Math Fluency probes online

http://www.interventioncentral.org

Examples of Early Math Fluency (Number Sense) CBM Probes

Quantity Discrimination

4	12

Missing Number

14	___	16	17

Number Identification

34	37	50	38	1
3	24	35	15	40

Sources: Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. School Psychology Review, 33, 234–248.

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. Assessment For Effective Intervention, 30(2), 3-14

Help

Bookmarks 884 blocked Check AutoLink AutoFill Send to

M Early Math Fluenc...

numberfly

The application to create CBM Early Math Fluency probes online

Quantity Discrimination (QD)

Description: The student is given a sheet of number pairs and must verbally identify the larger of the two values for each pair.

Select the ***lowest*** and ***highest*** numbers to be selected in the quantity-discrimination items:

FROM 0 TO 20

How many quantify discrimination items should appear ***in each row***?:

3 items

How many ***rows*** of items should appear on the student worksheet?:

8 Submit

QD Directions: Download directions for administering and scoring *Quantity Discrimination* probes, test statistics, & brief guidelines for use in an **RTI process**

QD Graph: Access a time-series graph to chart student progress using *Quantity Discrimination* probes

Missing Number (MN)

Description: The student is given a sheet that contains a series of 3- or 4-number sequences. In each sequence, one number is missing.

Early Math Fluency Probes: To Measure Student 'Number Sense': Grades K-1			
Number Identification (1 minute probe)			
Grade/Percentile	**Fall**	**Winter**	**Spring**
Kindergarten/50th percentile	31	48	55
Grade 1/50th percentile	43	57	62
Quantity Discrimination (1 minute probe)			
Grade/Percentile	**Fall**	**Winter**	**Spring**
Kindergarten/50th percentile	10	19	27
Grade 1/50th percentile	22	31	34
Missing Number (1 minute probe)			
Grade/Percentile	**Fall**	**Winter**	**Spring**
Kindergarten/50th percentile	04	11	15
Grade 1/50th percentile	12	17	19

Source: AIMSweb. (2006). *Growth table multi-year aggregate 2006-2007 school year*. Retrieved on September 23, 2011, from http://www.miu4.k12.pa.us/textfiles/datatools/IU%20norms.pdf.

NOTE: The original growth table from AIMSweb has percentile rankings for 10-90%iles.

Assessing Basic Academic Skills: Curriculum-Based Measurement

Mathematics: Single-skill basic arithmetic combinations are an ‘adequate measure of performance’ for low-achieving elementary & middle school students.

Websites to create CBM math computation probes:

- *www.interventioncentral.org*

```
      49 r31
47/2334
  -188
    454
   -423
     31
```

- *www.superkids.com*

$$\frac{9}{18} \times \frac{19}{20} = \frac{19}{40}$$

Source: Espin, C. A., & Tindal, G. (1998). Curriculum-based measurement for secondary students. In M. R. Shinn (Ed.) *Advanced applications of curriculum-based measurement.* New York: Guilford Press.

CBM Math Computation Sample Goals

- Addition: Add two one-digit numbers: sums to 18
- Addition: Add 3-digit to 3-digit with regrouping from ones column only

- Subtraction: Subtract 1-digit from 2-digit with no regrouping
- Subtraction: Subtract 2-digit from 3-digit with regrouping from ones and tens columns

- Multiplication: Multiply 2-digit by 2-digit-no regrouping
- Multiplication: Multiply 2-digit by 2-digit with regrouping

CBM Math Computation Assessment: Preparation

- Select either single-skill or multiple-skill math probe format.
- Create student math computation worksheet (including enough problems to keep most students busy for 2 minutes)
- Create answer key

CBM Math Computation Assessment: Preparation

- Advantage of single-skill probes:
  - Can yield a more ‘pure’ measure of student’s computational fluency on a particular problem type

CBM Math Computation Assessment: Preparation

- Advantage of multiple-skill probes:
  - Allow examiner to gauge student's adaptability *between* problem types (e.g., distinguishing operation signs for addition, multiplication problems)
  - Useful for including previously learned computation problems to ensure that students retain knowledge.

Curriculum-Based Assessment Mathematics
Multiple-Skills Computation Probe: Examiner Copy

Item 1:
4 CD/4 CD Total
ADDITION: Three to five 3-digit numbers: Regrouping in any column

```
  663
 +208
 +628
 +411
 1910
```

Item 2:
16 CD/20 CD Total
DIVISION: 4-digit number divided by 2-digit number: no remainder

```
      193
  23/4439
    -23
     213
    -207
       69
      -69
        0
```

Item 3:
14 CD/34 CD Total
MULTIPLICATION: 4-digit number times 2-digit number: no regrouping

```
   2213
  x  12
   4426
  2213-
 26,556
```

Item 4:
5 CD/39 CD Total
SUBTRACTION: 5-digit number from a 5-digit number: regrouping from 1's & 10's columns

```
   36,841
 - 15,765
   21,076
```

Item 5:
4 CD/43 CD Total
ADDITION: Three to five 3-digit numbers: Regrouping in any column

```
  290
 +731
 +672
 1693
```

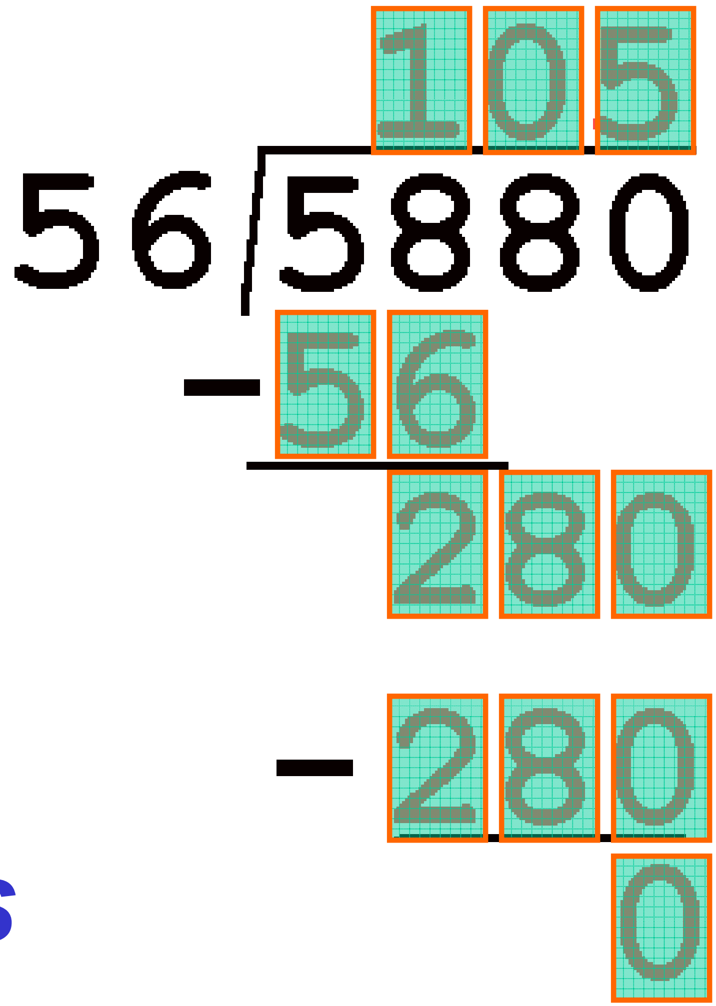
Item 6:
12 CD/55 CD Total
DIVISION: 4-digit number divided by 2-digit number: no remainder

```
       22
  68/1496
    -136
      136
     -136
        0
```

CBM Math Computation Assessment: Scoring

Unlike more traditional methods for scoring math computation problems, CBM gives the student credit for *each correct digit* in the answer. This approach to scoring is more sensitive to short-term student gains and acknowledges the child's partial competencies in math.

Math Computation: Scoring Examples



12 CDs

Math Computation: Scoring

Numbers Above Line Are Not Counted

Placeholders Are Counted

```
    1
    48
x   32
------
    96
  1440
------
  1536
```

Curriculum-Based Measurement: Math Computation (Adapted from Deno & Mirkin, 1977)		
Grade	Digits Correct in 2 Minutes	Digits Incorrect in 2 Minutes
1-3	20-38	6-14
4 & Up	40-78	6-14

CBM Math Computation Norms (Fuchs, Fuchs, Hintze, & Lembke, 2007)		
Grade	Assessment Time	End of Year Benchmark
Kindergarten	—	—
Grade 1	2 minutes	20 digits
Grade 2	2 minutes	20 digits
Grade 3	3 minutes	30 digits
Grade 4	3 minutes	40 digits
Grade 5	5 minutes	30 digits
Grade 6	6 minutes	35 digits

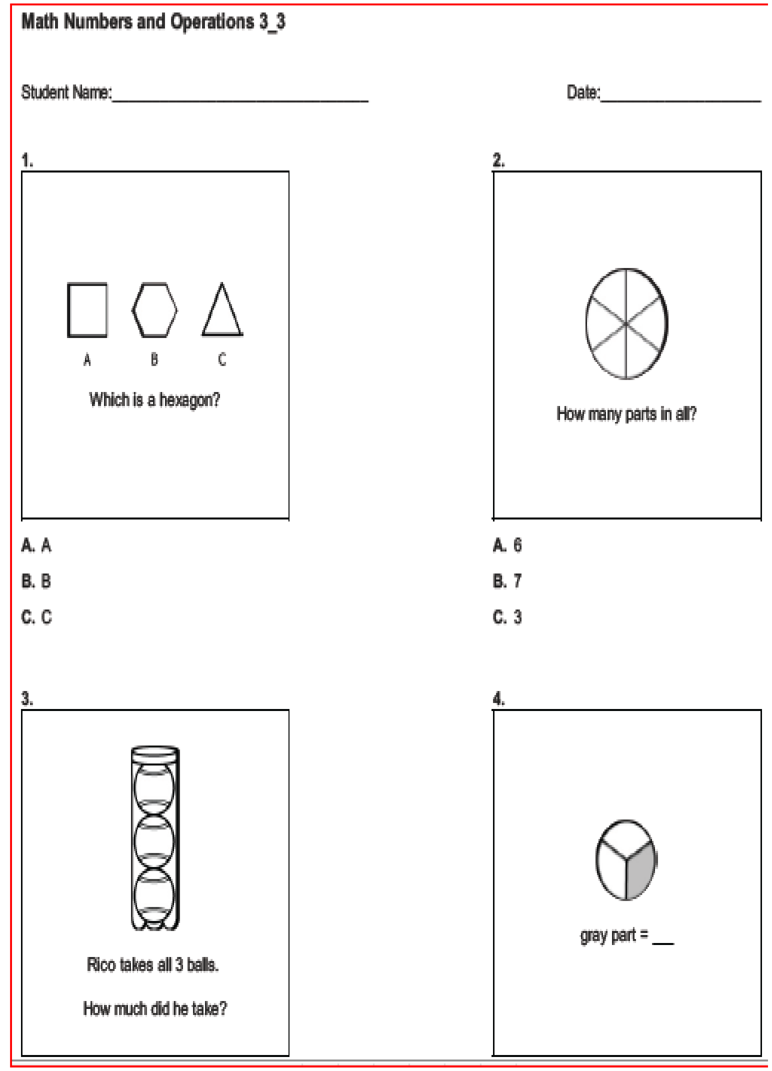
Source: Fuchs, D., Fuchs, L., Hintze, J., & Lembke, E. (2007). *Using curriculum-based measurement to determine response to intervention*. National Center on Student Progress Monitoring. Retrieved from http://www.studentprogress.org/summer_institute

Question: How can a school use CBM Math Computation probes if students are encouraged to use one of several methods to solve a computation problem—and have no fixed algorithm?

Answer: Students should know their 'math facts' automatically. Therefore, students can be given math computation probes to assess the speed and fluency of basic math facts—even if their curriculum encourages a variety of methods for solving math computation problems.

Math: Concepts & Applications

- The Math Concepts and Applications assessments (www.easycbm.com) were developed using as a guide the Math Curriculum Focal Points from the National Council of Teachers of Mathematics (NCTM). They include items sampling:
  - Algebra
  - Geometry & Measurement
  - Data Analysis & Number Operations


Math Numbers and Operations 3_3

Student Name:______________________ Date:______________

1.

A B C

Which is a hexagon?

A. A
B. B
C. C

2.

How many parts in all?

A. 6
B. 7
C. 3

3.

Rico takes all 3 balls.

How much did he take?

4.

gray part = __

Math Vocabulary Probes

Curriculum-Based Evaluation: Math Vocabulary

Format Option 1

- 20 vocabulary terms appear alphabetically in the right column. Items are drawn randomly from a ‘vocabulary pool’
- Randomly arranged definitions appear in the left column.
- The student writes the letter of the correct term next to each matching definition.
- The student receives 1 point for each correct response.
- Each probe lasts 5 minutes.
- 2-3 probes are given in a session.

Name: ___________________ Date: _______________________

Directions: Next to each definition in the left column, write the letter of the term from the right column that matches it.

Definition	**Term**
_B_ **1.** A property of some sets and binary operations in which the order of the objects is not important.	**A.** binary operation
___ **2.** Combines two objects of one type to form another object of the same type.	**B.** commutativity
___ **3.** A function in which the output changes in a reciprocal relationship to the input. In other words, if the input doubles, the output is cut in half.	**C.** Integer
___ **4.** Any number which is a counting number, zero, or the opposite of a counting number.	**D.** inverse proportion

Source: Howell, K. W. (2008). Best practices in curriculum-based evaluation and advanced reading. In A. Thomas & J. Grimes (Eds.), Best practices in school psychology V (pp. 397-418).

Curriculum-Based Evaluation: Math Vocabulary

Format Option 2

- 20 randomly arranged vocabulary definitions appear in the right column. Items are drawn randomly from a 'vocabulary pool'
- The student writes the name of the correct term next to each matching definition.
- The student is given 0.5 point for each correct term and another 0.5 point if the term is spelled correctly.
- Each probe lasts 5 minutes.
- 2-3 probes are given in a session.

Name: ____________________ Date: ______________________

Directions: Write the name of the term that completes each math definition.

Term	**Definition**
______________________	**1.** A property of some sets and binary operations in which the order of the objects is not important.
______________________	**2.** Combines two objects of one type to form another object of the same type.
______________________	**3.** A function in which the output changes in a reciprocal relationship to the input. In other words, if the input doubles, the output is cut in half.
______________________	**4.** Any number which is a counting number, zero, or the opposite of a counting number.

Source: Howell, K. W. (2008). Best practices in curriculum-based evaluation and advanced reading. In A. Thomas & J. Grimes (Eds.), Best practices in school psychology V (pp. 397-418).

How Does a Secondary School Determine a Student's Math Competencies?

"Tests [to assess secondary students' math knowledge] should be used or if necessary developed that measure students' procedural fluency as well as their conceptual understanding. Items should range in difficulty from simple applications of the algorithm to more complex. A variety of problem types can be used across assessments to tap students' conceptual knowledge." p. 469

Source: Ketterlin-Geller, L. R., Baker, S. K., & Chard, D. J. (2008). Best practices in mathematics instruction and assessment in secondary settings. In A. Thomas & J. Grimes (Eds.), Best practices in school psychology V (pp.465-475).

Discrete Categorization: A Strategy for Assessing Complex, Multi-Step Student Academic Tasks

Definition of *Discrete Categorization:* 'Listing a number of behaviors and checking off whether they were performed.' (Kazdin, 1989, p. 59).

- Approach allows educators to define a larger 'behavioral' goal for a student and to break that goal down into sub-tasks. (Each sub-task should be defined in such a way that it can be scored as 'successfully accomplished' or 'not accomplished'.)
- The constituent behaviors that make up the larger behavioral goal need not be directly related to each other. For example, 'completed homework' may include as sub-tasks 'wrote down homework assignment correctly' and 'created a work plan before starting homework'

Source: Kazdin, A. E. (1989). *Behavior modification in applied settings* (4th ed.). Pacific Gove, CA: Brooks/Cole..

Discrete Categorization Example: Math Study Skills

General Academic Goal: *Improve Tina's Math Study Skills*

Tina was struggling in her mathematics course because of poor study skills. The RTI Team and math teacher analyzed Tina's math study skills and decided that, to study effectively, she needed to:

- ❑ Check her math notes daily for completeness.
- ❑ Review her math notes daily.
- ❑ Start her math homework in a structured school setting.
- ❑ Use a highlighter and 'margin notes' to mark questions or areas of confusion in her notes or on the daily assignment.
- ❑ Spend sufficient 'seat time' at home each day completing homework.
- ❑ Regularly ask math questions of her teacher.

Discrete Categorization Example: Math Study Skills

General Academic Goal: *Improve Tina's Math Study Skills*

The RTI Team—with student and math teacher input—created the following intervention plan. The student Tina will:

- ❑ Obtain a copy of class notes from the teacher at the end of each class.
- ❑ Check her daily math notes for completeness against a set of teacher notes in 5th period study hall.
- ❑ Review her math notes in 5th period study hall.
- ❑ Start her math homework in 5th period study hall.
- ❑ Use a highlighter and 'margin notes' to mark questions or areas of confusion in her notes or on the daily assignment.
- ❑ Enter into her 'homework log' the amount of time spent that evening doing homework and noted any questions or areas of confusion.
- ❑ Stop by the math teacher's classroom during help periods (T & Th only) to ask highlighted questions (or to verify that Tina understood that week's instructional content) and to review the homework log.

Discrete Categorization Example: Math Study Skills

Academic Goal: *Improve Tina's Math Study Skills*

General measures of the success of this intervention include (1) rate of homework completion and (2) quiz & test grades.

To *measure treatment fidelity (*Tina's follow-through with sub-tasks of the checklist), the following strategies are used :

- ❑ Approached the teacher for copy of class notes. *Teacher observation.*
- ❑ Checked her daily math notes for completeness; reviewed math notes, started math homework in 5th period study hall. *Student work products; random spot check by study hall supervisor.*
- ❑ Used a highlighter and 'margin notes' to mark questions or areas of confusion in her notes or on the daily assignment. *Review of notes by teacher during T/Th drop-in period.*
- ❑ Entered into her 'homework log' the amount of time spent that evening doing homework and noted any questions or areas of confusion. *Log reviewed by teacher during T/Th drop-in period.*
- ❑ Stopped by the math teacher's classroom during help periods (T & Th only) to ask highlighted questions (or to verify that Tina understood that week's instructional content). *Teacher observation; student sign-in.*

‘Academic Enabler’ Observational Checklists: Measuring Students’ Ability to Manage Their Own Learning

‘Academic Enabler’ Skills: Why Are They Important? (Cont.)

Because academic enablers are often described as broad skill sets, however, they can be challenging to define in clear, specific, measureable terms. A useful method for defining a global academic enabling skill is to break it down into a checklist of component sub-skills--a process known as ‘discrete categorization’ (Kazdin, 1989). An observer can then use the checklist to note whether a student successfully displays each of the sub-skills.

Source: Kazdin, A. E. (1989). *Behavior modification in applied settings* (4th ed.). Pacific Gove, CA: Brooks/Cole.

'Academic Enabler' Skills: Why Are They Important? (Cont.)

Observational checklists that define academic enabling skills have several uses in Response to Intervention:

- Classroom teachers can use these skills checklists as convenient tools to assess whether a student possesses the minimum 'starter set' of academic enabling skills needed for classroom success.
- Teachers or tutors can share examples of academic-enabler skills checklists with students, training them in each of the sub-skills and encouraging them to use the checklists independently to take greater responsibility for their own learning.
- Teachers or other observers can use the academic enabler checklists periodically to monitor student progress during interventions--assessing formatively whether the student is using more of the sub-skills.

Source: Kazdin, A. E. (1989). *Behavior modification in applied settings* (4th ed.). Pacific Gove, CA: Brooks/Cole.

'Academic Enabler' Skills: Sample Observational Checklists

Study Skills. The student:	
❑ takes complete, organized class notes in legible form and maintains them in one accessible note book	Poor 1 Fair 2 Good 3 NA _
❑ reviews class notes frequently (e.g., after each class) to ensure understanding	Poor 1 Fair 2 Good 3 NA _
❑ When reviewing notes, uses highlighters, margin notes, or other strategies to note questions or areas of confusion for later review with teacher or tutor	Poor 1 Fair 2 Good 3 NA _
❑ follows an efficient strategy to study for tests and quizzes	Poor 1 Fair 2 Good 3 NA _
❑ allocates enough time to study for tests and quizzes	Poor 1 Fair 2 Good 3 NA _
❑ is willing to seek help from the teacher to answer questions or clear up areas of confusion	Poor 1 Fair 2 Good 3 NA _
❑ Other: ____	

Comments:

____

____

'Academic Enabler' Skills: Sample Observational Checklists

Organization Skills. The student:				
❑ arrives to class on time.	Poor 1	Fair 2	Good 3	NA _
❑ maintains organization of locker to allow student to efficiently store and retrieve needed books, assignments, work materials, and personal belongings	Poor 1	Fair 2	Good 3	NA _
❑ maintains organization of backpack or book bag to allow student to efficiently store and retrieve needed books, assignments, work materials, and personal belongings	Poor 1	Fair 2	Good 3	NA _
❑ brings to class the necessary work materials expected for the course (e.g., pen, paper, calculator, etc.)	Poor 1	Fair 2	Good 3	NA _
❑ is efficient in switching work materials when transitioning from one in-class learning activity to another	Poor 1	Fair 2	Good 3	NA _
❑ Other: ________________________________	Poor 1	Fair 2	Good 3	NA _
Comments: ________________________________ ________________________________				

'Academic Enabler' Skills: Sample Observational Checklists

Homework Completion. The student:	
❑ writes down homework assignments accurately and completely	Poor 1 Fair 2 Good 3 NA _
❑ makes use of available time in school (e.g., study halls, homeroom) to work on homework	Poor 1 Fair 2 Good 3 NA _
❑ has an organized, non-distracting workspace available at home to do homework	Poor 1 Fair 2 Good 3 NA _
❑ creates a work plan before starting homework (e.g., sequencing the order in which assignments are to be completed; selecting the most challenging assignment to start first when energy and concentration are highest)	Poor 1 Fair 2 Good 3 NA _
❑ when completing homework, uses highlighters, margin notes, or other strategies to note questions or areas of confusion for later review with teacher or tutor	Poor 1 Fair 2 Good 3 NA _
❑ turns in homework on time	Poor 1 Fair 2 Good 3 NA _
❑ Other: ______________________________	Poor 1 Fair 2 Good 3 NA _
Comments: ______________________________ ______________________________	

‘Academic Enabler’ Skills: Sample Observational Checklists

Cooperative Learning Skills. The student:				
❑ participates in class discussion	Poor 1	Fair 2	Good 3	NA _
❑ gets along with others during group/pair activities	Poor 1	Fair 2	Good 3	NA _
❑ participates fully in group/pair activities	Poor 1	Fair 2	Good 3	NA _
❑ does his or her ‘fair share’ of work during group/pair activities	Poor 1	Fair 2	Good 3	NA _
❑ is willing to take a leadership position during group/pair activities	Poor 1	Fair 2	Good 3	NA _
❑ Other: ____________________	Poor 1	Fair 2	Good 3	NA _

Comments:

____

____

'Academic Enabler' Skills: Sample Observational Checklists

Independent Seat Work. The student:	
❑ has necessary work materials for the assignment	Poor 1 Fair 2 Good 3 NA _
❑ is on-task during the assignment at a level typical for students in the class	Poor 1 Fair 2 Good 3 NA _
❑ refrains from distracting behaviors (e.g., talking with peers without permission, pen tapping, vocalizations such as loud sighs or mumbling, etc.)	Poor 1 Fair 2 Good 3 NA _
❑ recognizes when he or she needs teacher assistance and is willing to that assistance	Poor 1 Fair 2 Good 3 NA _
❑ requests teacher assistance in an appropriate manner	Poor 1 Fair 2 Good 3 NA _
❑ requests assistance from the teacher only when really needed	Poor 1 Fair 2 Good 3 NA _
❑ if finished with the independent assignment before time expires, uses remaining time to check work or engage in other academic activity allowed by teacher	Poor 1 Fair 2 Good 3 NA _
❑ takes care in completing work—as evidenced by the quality of the finished assignment	Poor 1 Fair 2 Good 3 NA _
❑ is reliable in turning in assignments done in class.	Poor 1 Fair 2 Good 3 NA _
❑ Other: ______________________________	Poor 1 Fair 2 Good 3 NA _

Comments:

____

____

'Academic Enabler' Skills: Sample Observational Checklists

Motivation. The student:	
❑ has a positive sense of 'self-efficacy' about the academic content area (self-efficacy can be defined as the confidence that one can be successful in the academic discipline or subject matter if one puts forth reasonable effort)	Poor 1 Fair 2 Good 3 NA –
❑ displays some apparent *intrinsic* motivation to engage in course work (e.g., is motivated by topics and subject matter discussed or covered in the course; finds the act of working on course assignments to be reinforcing in its own right)	Poor 1 Fair 2 Good 3 NA –
❑ displays apparent *extrinsic* motivation to engage in course work (e.g., is motivated by grades, praise, public recognition of achievement, access to privileges such as sports eligibility, or other rewarding outcomes)	Poor 1 Fair 2 Good 3 NA –
❑ Other: ______________________________	Poor 1 Fair 2 Good 3 NA –
Comments: ______________________________ ______________________________	

'Academic Enabler' Skills: Sample Observational Checklists

Teacher-Defined Academic Enabling Skill:

Skill Name: ______________________________

Essential Subskills: The student::

❑ ______________________________ ______________________________ ______________________________	Poor 1 · Fair 2 · Good 3 · NA –
❑ ______________________________ ______________________________ ______________________________	Poor 1 · Fair 2 · Good 3 · NA –
❑ ______________________________ ______________________________ ______________________________	Poor 1 · Fair 2 · Good 3 · NA –
❑ ______________________________ ______________________________ ______________________________	Poor 1 · Fair 2 · Good 3 · NA
❑ ______________________________ ______________________________ ______________________________	Poor 1 · Fair 2 · Good 3 · NA

Comments:

____

____

Team Activity: Using Formative Math Assessments to Inform Instruction/Intervention

- Review the free math measures just discussed for screening and monitoring the progress of students in math skills:
  - CBM: Early Math Fluency (Number Sense)
  - CBM: Math Computation Fluency: Math Facts
  - CBM: Math Concepts & Applications
  - CBM: Math Vocabulary
  - Checklist: Cognitive Strategies
- Have a discussion about how you or others in your school might explore/pilot/use these or similar math assessment tools.

Elementary Tier 1 Intervention: Case Example: John: Math Computation

Jim Wright
www.interventioncentral.org

Case Example: Math Computation

The Problem

- John is a fourth-grade student. His teacher, Mrs. Kennedy, is concerned that John appears to be much slower in completing math computation items than are his classmates.

Case Example: Math Computation

Core Instruction

- John's school uses the *Everyday Math* curriculum (McGraw Hill/University of Chicago). In addition to the basic curriculum the series contains intervention exercises for students who need additional practice or remediation.

  As an extension of core instruction, his teacher works with a small group of children in her room—including John– having them complete these practice exercises to boost their math computation fluency. While other children in this group appear to benefit from the assistance, John does not make noticeable gains in his computation speed.

Case Example: Math Computation

The Evidence

- Mrs. Kennedy collects and reviews information that may be relevant to understanding John's math computation concern:

  Teacher Interview. Ms. Kennedy talks with John's Grade 3 teacher from last year who confirms that John was slow in completing math facts in that setting as well—but was accurate in his work and appeared motivated to do computation assignments.

Case Example: Math Computation

The Evidence (Cont.)

- *Review of Records.* When Mrs. Kennedy reviews John's past report cards and other records from his cumulative file, she does not find any comments or other evidence that he displayed fine-motor delays that might interfere with computation fluency.

- *Work Products*. Mrs. Kennedy reviews examples of John's work on untimed math computation worksheets. Similar to observations shared by the 3rd grade teacher, Mrs. Kennedy notes that John's work is accurate—even though he did not complete as many problems as peers.

Case Example: Math Computation

The Evidence (Cont.)

- *Direct Observation.* Watching John complete a computation worksheet, his teacher notes that John counts on his fingers. This appears to slow down his computation speed considerably.

Case Example: Math Computation

The Intervention

- Mrs. Kennedy met with a consultant to create a Tier 1 (classroom) intervention plan for John. Both the consultant and teacher agreed that John was slow in math computation because he relied on finger counting to compute number problems rather than using the more efficient strategies of mental arithmetic and automatic recall of math facts.

Case Example: Math Computation

The Intervention (Cont.)

- Mrs. Kennedy decided to institute a version of math computation time-drills as a technique to boost John's computation speed and (she hoped) encourage him to give up the finger-counting habit.

  Each day, John would self-administer and score 3 separate three-minute time drills using multiplication facts….

Math Intervention: Tier I or II: Elementary & Secondary: *Self-Administered 'Math Fact' Timed Drills With Performance Self-Monitoring & Incentives*

1. The student is given a math computation worksheet of a specific problem type, along with an answer key [Academic Opportunity to Respond].
2. The student consults his or her performance chart and notes previous performance. The student is encouraged to try to 'beat' his or her most recent score.
3. The student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems as possible. The student sets a timer and works on the computation sheet until the timer rings. [Active Student Responding]
4. The student checks his or her work, giving credit for each ***correct digit*** (digit of correct value appearing in the correct place-position in the answer). [Performance Feedback]
5. The student records the day's score of TOTAL number of correct digits on his or her personal performance chart.
6. The student receives praise or a reward if he or she exceeds the most recently posted number of correct digits.

Application of 'Learn Unit' framework from : *Heward, W.L. (1996). Three low-tech strategies for increasing the frequency of active student response during group instruction. In R. Gardner, D. M.S ainato, J. O. Cooper, T. E. Heron, W. L. Heward, J. W. Eshleman,& T. A. Grossi (Eds.), Behavior analysis in education: Focus on measurably superior instruction (pp.283-320). Pacific Grove, CA:Brooks/Cole.*

Self-Administered Arithmetic Combination Drills: Examples of Student Worksheet and Answer Key

Curriculum-Based Assessment Mathematics
Single-Skill Computation Probe: Student Copy

Student: Date: ________________

8 x6	3 x8	2 x3
9 x5	6 x5	1 x3
3 x8	3 x7	2 x4

www.interventioncentral.org

Curriculum-Based Assessment Mathematics
Single-Skill Computation Probe: Examiner Copy

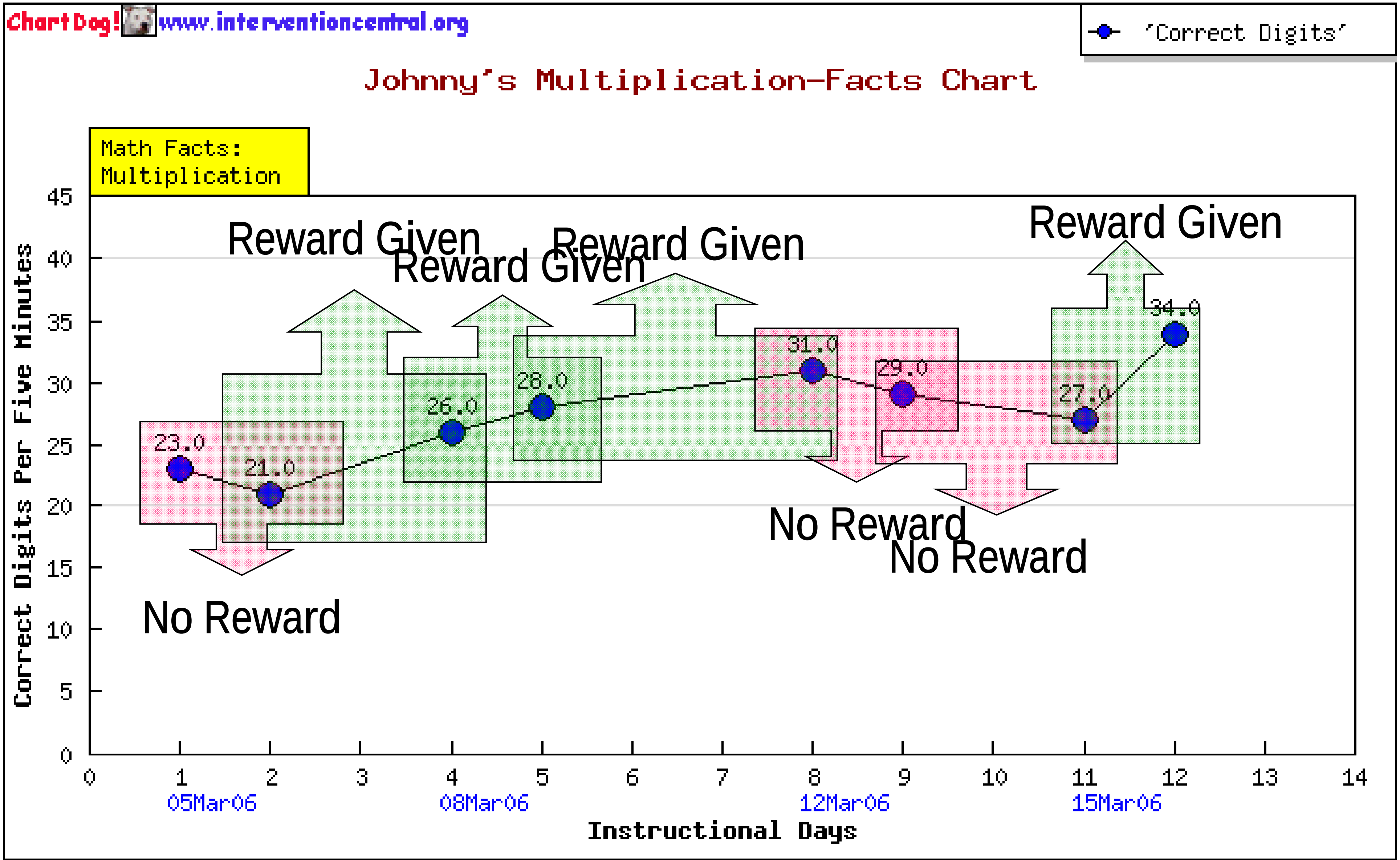
MULTIPLICATION: Multiplication facts: 0 to 9

8 x 6 48	3 x 8 24	2 x 3 6
9 x 5 45	6 x 5 30	1 x 3 3
3 x 8 24	3 x 7 21	2 x 4 8

www.interventioncentral.org
Click for Student Worksheet

Worksheets created using Math Worksheet Generator. Available online at:
http://www.interventioncentral.org/htmdocs/tools/mathprobe/addsing.php

Self-Administered Arithmetic Combination Drills…



Case Example: Math Computation

Documentation and Goal-Setting

- While meeting with the consultant, Mrs. Kennedy filled out a Tier 1 intervention plan for the student. On the plan, she listed interventions to be used, a checkup date (5 instructional weeks), and data to be used to assess student progress.
- Mrs. Kennedy decided to monitor John's computation progress once per week using a 2-minute curriculum-based measurement math computation probe.

- Progress-monitoring worksheets were created to assess John using the Math Computation Probe Generator on Intervention Central (*www.interventioncentral.org*).

Example of Math Computation Probe: Answer Key

MULTIPLICATION: 2-digit number times 2-digit number: no regrouping

Item 1:
8 CD/8 CD Total

```
   12
 x 23
   36
  24-
  276
```

Item 2:
8 CD/16 CD Total

```
   43
 x 22
   86
  86-
  946
```

Item 4:
8 CD/32 CD Total

```
   30
 x 21
   30
  60-
  630
```

Item 5:
8 CD/40 CD Total

```
   30
 x 21
   30
  60-
  630
```

Case Example: Math Computation

Goal-Setting

- Mrs. Kennedy's school used math computation guidelines that indicated that defined fluency in math computation at 40 correct digits (CDs) or more in two minutes.
- At baseline, John was found to calculate an average of 18 CDs per 2 minutes.
- Mrs. Kennedy decided to set a goal of 2 additional CDs per week. Her intermediate goal was for John to compute at least 28 CDs per 2 minutes at the end of five weeks.

Curriculum-Based Measurement: Math Computation (Adapted from Deno & Mirkin, 1977)		
Grade	**Digits Correct in 2 Minutes**	**Digits Incorrect in 2 Minutes**
1-3	20-38	6-14
4 & Up	40-78	6-14
Comments: These math computation norms are still widely referenced. However, the norms were collected nearly 30 years ago and may not be widely representative because they were drawn from a relatively small sample of students. Additionally, the norms make no distinction between easy and more challenging math computation problem types. Because of these limitations, these norms are best regarded as a rough indicator of 'typical' student math computation skills.		

Tier 1/Classroom Intervention Planning Sheet

Teacher/Team: _________Mrs. Kenndy___________ Date: _____Oct 10, 2010_____ Student: ________John S.____________

Student Problem Definition #1: Slow math computation speed (computes multiplication facts at about half the expected rate)___

Student Problem Definition #2: ____

[Optional] Person(s) assisting with intervention planning process: ______Angela Cordone, Special Education Teacher_________

Interventions: Essential Elements (Witt et al., 2004)

- Clear problem-definition(s)
- Baseline data
- Goal for improvement
- Progress-monitoring plan

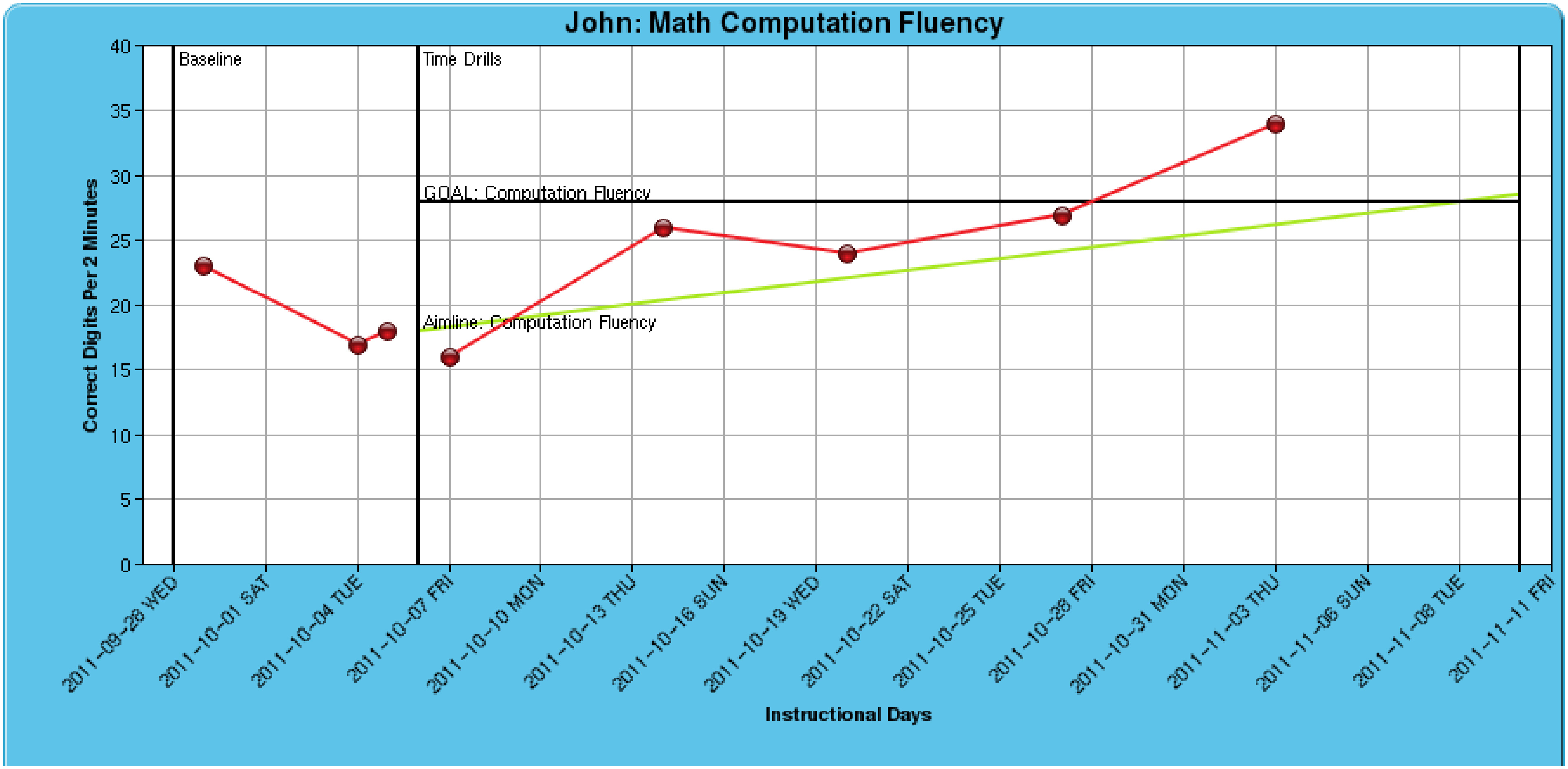
Intervention Description	**Intervention Delivery**	**Check-Up Date**	**Assessment Data**	
Describe each intervention that you plan to use to address the student's concern(s).	List key details about delivery of the intervention, such as:; (1) where & when the intervention will be used; (2) the adult-to-student ratio; (3) how frequently the intervention will take place; (4) the length of time each session of the intervention will last;.	Select a date when the data will be reviewed to evaluate the intervention.	Note what classroom data will be used to establish baseline, set a goal for improvement, and track the student's progress during this intervention.	
Math Computation Timed Drill (see attached description)	**Use math worksheet generator on www.interventioncentral.org to create all time-drill and assessment materials.**	**Five instructional weeks.**	Type(s) of Data to Be Used: **CBM Math Computation: multiplication facts**	
			Baseline	Goal by Check-Up
			18 CDs in 2 mins	**28 CDs in 2 mins**

Case Example: Math Computation

The Outcome

- When the intervention had been in place for 5 weeks, Mrs. Kennedy found that John had exceeded his intermediate goal of 28 CDs per 2 minutes—the actual number was 34 CDs.
- Mrs. Kennedy judged that the intervention was effective. She decided to continue the intervention without changes for another five weeks with the expectation that John would reach his goal (40 CDs in 2 minutes) by that time.

Response to Intervention



Case Example: Math Computation

The Outcome (Alternate Ending)

- When the intervention had been in place for 5 weeks, Mrs. Kennedy found that John had not attained his intermediate goal of 28 CDs in 2 minutes. In fact, he had made virtually no progress.
- Mrs. Kennedy decided to try other strategies in the classroom to help John to acquire math facts (e.g., Cover-Copy-Compare).
- However, Mrs. Kennedy also referred the student for additional RTI support (Tier 2 or 3).