RTI: Best Practices in Mathematics Interventions

Jim Wright

www.interventioncentral.org
Response to Intervention

Workshop PPTs and Handout Available at:

http://www.jimwrightonline.com/SCBOCES.php
Response to Intervention

Workshop Agenda

RTI & Math Interventions: An Overview

Math: Computational Fluency

Math: Applied Problems

Math: Progress-Monitoring Tools

Web Resources to Support Math Assessment & Interventions

www.interventioncentral.org
Essential Elements of RTI  (Fairbanks, Sugai, Guardino, & Lathrop, 2007)

1. A “continuum of evidence-based services available to all students” that range from universal to highly individualized & intensive

2. “Decision points to determine if students are performing significantly below the level of their peers in academic and social behavior domains”

3. “Ongoing monitoring of student progress”

4. “Employment of more intensive or different interventions when students do not improve in response” to lesser interventions

5. “Evaluation for special education services if students do not respond to intervention instruction”

RTI ‘Pyramid of Interventions’

**Tier 1: Universal interventions.**
Available to all students in a classroom or school. Can consist of whole-group or individual strategies or supports.

**Tier 2: Individualized interventions.**
Subset of students receive interventions targeting specific needs.

**Tier 3: Intensive interventions.**
Students who are ‘non-responders’ to Tiers 1 & 2 are referred to the RTI Team for more intensive interventions.
The Key Role of Classroom Teachers in RTI: 6 Steps

1. The teacher defines the student academic or behavioral problem clearly.

2. The teacher decides on the best explanation for why the problem is occurring.

3. The teacher selects ‘evidence-based’ interventions.

4. The teacher documents the student’s Tier 1 intervention plan.

5. The teacher monitors the student’s response (progress) to the intervention plan.

6. The teacher knows what the next steps are when a student fails to make adequate progress with Tier 1 interventions alone.
Response to Intervention

Avg Classroom Academic Performance Level

Discrepancy 1: Skill Gap (Current Performance Level)

Target Student

Discrepancy 2: Gap in Rate of Learning ('Slope of Improvement')

'Dual-Discrepancy': RTI Model of Learning Disability (Fuchs 2003)

www.interventioncentral.org
### Response to Intervention

#### Scheduling Elementary Tier 2 Interventions

**Option 3: ‘Floating RTI’: Gradewide Shared Schedule.** Each grade has a scheduled RTI time across classrooms. No two grades share the same RTI time. Advantages are that outside providers can move from grade to grade providing push-in or pull-out services and that students can be grouped by need across different teachers within the grade.

#### Anyplace Elementary School: RTI Daily Schedule

<table>
<thead>
<tr>
<th>Grade</th>
<th>Classroom 1</th>
<th>Classroom 2</th>
<th>Classroom 3</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Classroom 1</td>
<td>Classroom 2</td>
<td>Classroom 3</td>
<td>9:00-9:30</td>
</tr>
<tr>
<td>1</td>
<td>Classroom 1</td>
<td>Classroom 2</td>
<td>Classroom 3</td>
<td>9:45-10:15</td>
</tr>
<tr>
<td>2</td>
<td>Classroom 1</td>
<td>Classroom 2</td>
<td>Classroom 3</td>
<td>10:30-11:00</td>
</tr>
<tr>
<td>3</td>
<td>Classroom 1</td>
<td>Classroom 2</td>
<td>Classroom 3</td>
<td>12:30-1:00</td>
</tr>
<tr>
<td>4</td>
<td>Classroom 1</td>
<td>Classroom 2</td>
<td>Classroom 3</td>
<td>1:15-1:45</td>
</tr>
<tr>
<td>5</td>
<td>Classroom 1</td>
<td>Classroom 2</td>
<td>Classroom 3</td>
<td>2:00-2:30</td>
</tr>
</tbody>
</table>

‘Elbow Group’ Activity: What are common student mathematics concerns in your school?

In your ‘elbow groups’:

• Discuss the most common student mathematics problems that you encounter in your school(s). At what grade level do you typically encounter these problems?

• Be prepared to share your discussion points with the larger group.
Challenge: Defining Research-Based Principles of Effective Math Instruction & Intervention
Appropriate instruction begins with the core program that provides:

- high quality, research-based instruction to all students in the general education class provided by qualified teachers;
- differentiated instruction\(^1\) to meet the wide range of student needs;
- curriculum that is aligned to the State learning standards and grade level performance indicators for all general education subjects; and
- instructional strategies that utilize a formative assessment process.

It is recommended that schools use the New York State (NYS) curriculum guides to ensure that curriculum is aligned to NYS learning standards. These can be found at http://www.p12.nysed.gov/ciai/cores.html.

Appropriate instruction in mathematics includes instruction in problem-solving, arithmetic skill and fluency, conceptual knowledge/number sense and reasoning ability.

For additional information, see Foundations for Success: The Final Report of the National Mathematics Advisory Panel at http://www.ed.gov/about/bdscomm/list/mathpanel/index.html This report contains 45 findings and recommendations on curricular content, teachers and teacher education, instructional practices and materials, learning processes and assessments.

Additional resources for appropriate instruction in mathematics include, but are not limited to, the Institute of Education Sciences (IES) Practice Guide from What Works Clearinghouse, which offers eight recommendations for identifying and supporting students struggling in mathematics, intended to be implemented within an RtI framework and the guide “Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools” which can be found at http://ies.ed.gov/ncee/wwc/pdf/practiceguides/rti_math_pg_042109.pdf.

National Mathematics Advisory Panel Releases Final Report

On March 13, 2008, the National Mathematics Advisory Panel presented its Final Report to the President of the United States and the Secretary of Education. Copies of these ground-breaking reports, rich with information for parents, teachers, policy makers, the research community, and others, are provided below.


Final Report [PDF (851 KB)] [Word (1 MB)]

Draft Task Group Reports

- Conceptual Knowledge and Skills [Word (1.3 MB)]
- Learning Processes [Word (7.9 MB)]
- Instructional Practices [Word (2.9 MB)]
- Teachers [Word (1.2 MB)]
- Assessment [Word (876 KB)]

Draft Subcommittee Reports

- Standards of Evidence [PDF (68 KB)] [Word (276 KB)]
- Instructional Materials [Word (958 KB)]
- National Survey of Algebra Teachers for the National Math Panel [PDF (4.1 MB)] [Word (3.2 MB)]

Fact Sheet

Paper copies of these reports may be ordered at FDPubs.ed.gov.

If you need any of these documents in an alternative format, please contact the National Math Panel at NationalMathPanel@ed.gov.
Math Advisory Panel Report at:

http://www.ed.gov/mathpanel

• “The areas to be studied in mathematics from pre-kindergarten through eighth grade should be streamlined and a well-defined set of the most important topics should be emphasized in the early grades. Any approach that revisits topics year after year without bringing them to closure should be avoided.”

• “Proficiency with whole numbers, fractions, and certain aspects of geometry and measurement are the foundations for algebra. Of these, knowledge of fractions is the most important foundational skill not developed among American students.”

• “Conceptual understanding, computational and procedural fluency, and problem solving skills are equally important and mutually reinforce each other. Debates regarding the relative importance of each of these components of mathematics are misguided.”

• “Students should develop immediate recall of arithmetic facts to free the “working memory” for solving more complex problems.”

The Elements of Mathematical Proficiency: What the Experts Say...
5 Strands of Mathematical Proficiency

1. Understanding
2. Computing
3. Applying
4. Reasoning
5. Engagement


5 Big Ideas in Beginning Reading

1. Phonemic Awareness
2. Alphabetic Principle
3. Fluency with Text
4. Vocabulary
5. Comprehension

Response to Intervention

Five Strands of Mathematical Proficiency

1. **Understanding:** Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean.

2. **Computing:** Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.

3. **Applying:** Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.

Five Strands of Mathematical Proficiency (Cont.)

4. **Reasoning:** Using logic to explain and justify a solution to a problem or to extend from something known to something less known.

5. **Engaging:** Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.

**Response to Intervention**

**Table Activity: Evaluate Your School’s Math Proficiency…**

- As a group, review the National Research Council ‘Strands of Math Proficiency’.

- Which strand do you feel that your school / curriculum does the **best** job of helping students to attain proficiency?

- Which strand do you feel that your school / curriculum should put the **greatest effort** to figure out how to help students to attain proficiency?

- Be prepared to share your results.

**Five Strands of Mathematical Proficiency (NRC, 2002)**

1. **Understanding:** Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean.

2. **Computing:** Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.

3. **Applying:** Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.

4. **Reasoning:** Using logic to explain and justify a solution to a problem or to extend from something known to something less known.

5. **Engaging:** Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.
An RTI Challenge: Limited Research to Support Evidence-Based Math Interventions

“... in contrast to reading, core math programs that are supported by research, or that have been constructed according to clear research-based principles, are not easy to identify. Not only have exemplary core programs not been identified, but also there are no tools available that we know of that will help schools analyze core math programs to determine their alignment with clear research-based principles.” p. 459

Challenge:
Understanding the Student With ‘Math Difficulties’
Three General Levels of Math Skill Development
(Kroesbergen & Van Luit, 2003)

As students move from lower to higher grades, they move through levels of acquisition of math skills, to include:

- Number sense
- Basic math operations (i.e., addition, subtraction, multiplication, division)
- Problem-solving skills: "The solution of both verbal and nonverbal problems through the application of previously acquired information" (Kroesbergen & Van Luit, 2003, p. 98)

Who is At Risk for Poor Math Performance?: A Proactive Stance

“...we use the term mathematics difficulties rather than mathematics disabilities. Children who exhibit mathematics difficulties include those performing in the low average range (e.g., at or below the 35th percentile) as well as those performing well below average...Using higher percentile cutoffs increases the likelihood that young children who go on to have serious math problems will be picked up in the screening.” p. 295

Profile of Students with Math Difficulties
(Kroesbergen & Van Luit, 2003)

Although the group of students with difficulties in learning math is very heterogeneous, in general, these students have memory deficits leading to difficulties in the acquisition and remembering of math knowledge.

Moreover, they often show inadequate use of strategies for solving math tasks, caused by problems with the acquisition and the application of both cognitive and metacognitive strategies.

Because of these problems, they also show deficits in generalization and transfer of learned knowledge to new and unknown tasks.

Profile of Students With Significant Math Difficulties

1. **Spatial organization.** The student commits errors such as misaligning numbers in columns in a multiplication problem or confusing directionality in a subtraction problem (and subtracting the original number—minuend—from the figure to be subtracted (subtrahend)).

2. **Visual detail.** The student misreads a mathematical sign or leaves out a decimal or dollar sign in the answer.

3. **Procedural errors.** The student skips or adds a step in a computation sequence. Or the student misapplies a learned rule from one arithmetic procedure when completing another, different arithmetic procedure.

4. **Inability to ‘shift psychological set’.** The student does not shift from one operation type (e.g., addition) to another (e.g., multiplication) when warranted.

5. **Graphomotor.** The student’s poor handwriting can cause him or her to misread handwritten numbers, leading to errors in computation.

6. **Memory.** The student fails to remember a specific math fact needed to solve a problem. (The student may KNOW the math fact but not be able to recall it at ‘point of performance’.)

7. **Judgment and reasoning.** The student comes up with solutions to problems that are clearly unreasonable. However, the student is not able adequately to evaluate those responses to gauge whether they actually make sense in context.

Important elements of math instruction for low-performing students:

- “Providing teachers and students with data on student performance”
- “Using peers as tutors or instructional guides”
- “Providing clear, specific feedback to parents on their children’s mathematics success”
- “Using principles of explicit instruction in teaching math concepts and procedures.” p. 51

Team Activity: How Do Schools Implement Strategies to Reach Low-Performing Math Students?

At your table, review the instructional recommendations (Baker et al., 2002) for low-performing math students. How can your school promote implementation of these recommendations?

1. “Providing teachers and students with data on student performance”
2. “Using peers as tutors or instructional guides”
3. “Providing clear, specific feedback to parents on their children’s mathematics success”
4. “Using principles of explicit instruction in teaching math concepts and procedures.”
Challenge: Defining Early Numeracy Skills: Number Sense and Counting
What is ‘Number Sense’?
(Clarke & Shinn, 2004)

“...the ability to understand the meaning of numbers and define different relationships among numbers.

Children with number sense can recognize the relative size of numbers, use referents for measuring objects and events, and think and work with numbers in a flexible manner that treats numbers as a sensible system.” p. 236

What Are Stages of ‘Number Sense’?

(Berch, 2005, p. 336)

1. **Innate Number Sense.** Children appear to possess ‘hard-wired’ ability (neurological ‘foundation structures’) to acquire number sense. Children’s innate capabilities appear also to include the ability to ‘represent general amounts’, not specific quantities. This innate number sense seems to be characterized by skills at estimation (‘approximate numerical judgments’) and a counting system that can be described loosely as ‘1, 2, 3, 4, … a lot’.

2. **Acquired Number Sense.** Young students learn through indirect and direct instruction to count specific objects beyond four and to internalize a number line as a mental representation of those precise number values.

Response to Intervention

Task Analysis of Number Sense & Operations
(Methe & Riley-Tillman, 2008)

“Knowing the fundamental subject matter of early mathematics is critical, given the relatively young stage of its development and application... as well as the large numbers of students at risk for failure in mathematics. Evidence from the Early Childhood Longitudinal Study confirms the Matthew effect phenomenon, where students with early skills continue to prosper over the course of their education while children who struggle at kindergarten entry tend to experience great degrees of problems in mathematics. Given that assessment is the core of effective problem solving in foundational subject matter, much less is known about the specific building blocks and pinpoint subskills that lead to a numeric literacy, early numeracy, or number sense...” p. 30

Task Analysis of Number Sense & Operations
(Methe & Riley-Tillman, 2008)

1. Counting

2. Comparing and Ordering: Ability to compare relative amounts
   e.g., more or less than; ordinal numbers: e.g., first, second, third)

3. Equal partitioning: Dividing larger set of objects into ‘equal parts’

4. Composing and decomposing: Able to create different
   subgroupings of larger sets (for example, stating that a group of 10
   objects can be broken down into 6 objects and 4 objects or 3
   objects and 7 objects)

5. Grouping and place value: “abstractly grouping objects into sets
   of 10” (p. 32) in base-10 counting system.

6. Adding to/taking away: Ability to add and subtract amounts from
   sets “by using accurate strategies that do not rely on laborious
   enumeration, counting, or equal partitioning.” P. 32

Children’s Understanding of Counting Rules

The development of children’s counting ability depends upon the development of:

- **One-to-one correspondence**: “one and only one word tag, e.g., ‘one,’ ‘two,’ is assigned to each counted object”.
- **Stable order**: “the order of the word tags must be invariant across counted sets”.
- **Cardinality**: “the value of the final word tag represents the quantity of items in the counted set”.
- **Abstraction**: “objects of any kind can be collected together and counted”.
- **Order irrelevance**: “items within a given set can be tagged in any sequence”.

Challenge: Acquisition and Fluency in Math Computation
Benefits of Automaticity of ‘Arithmetic Combinations’
(Gersten, Jordan, & Flojo, 2005)

- There is a strong correlation between poor retrieval of arithmetic combinations (‘math facts’) and global math delays
- Automatic recall of arithmetic combinations frees up student ‘cognitive capacity’ to allow for understanding of higher-level problem-solving
- By internalizing numbers as mental constructs, students can manipulate those numbers in their head, allowing for the intuitive understanding of arithmetic properties, such as associative property and commutative property

How much is $3 + 8$?: Strategies to Solve... 

Least efficient strategy: Count out and group 3 objects; count out and group 8 objects; count all objects:

$$
\begin{array}{cccccccc}
\hline
\text{3 objects} & \text{+} & \text{8 objects} & \text{=11} \\
\hline
\end{array}
$$

More efficient strategy: Begin at the number 3 and ‘count up’ 8 more digits (often using fingers for counting):

$$3 + 8$$

More efficient strategy: Begin at the number 8 (larger number) and ‘count up’ 3 more digits:

$$8 + 3$$

Most efficient strategy: ‘3 + 8’ arithmetic combination is stored in memory and automatically retrieved: \textbf{Answer} = 11

Math Skills: Importance of Fluency in Basic Math Operations

“[A key step in math education is] to learn the four basic mathematical operations (i.e., addition, subtraction, multiplication, and division). Knowledge of these operations and a capacity to perform mental arithmetic play an important role in the development of children’s later math skills. Most children with math learning difficulties are unable to master the four basic operations before leaving elementary school and, thus, need special attention to acquire the skills. A … category of interventions is therefore aimed at the acquisition and automatization of basic math skills.”

Big Ideas: Learn Unit (Heward, 1996)

The three essential elements of effective student learning include:

1. **Academic Opportunity to Respond.** The student is presented with a meaningful opportunity to respond to an academic task. A question posed by the teacher, a math word problem, and a spelling item on an educational computer ‘Word Gobbler’ game could all be considered academic opportunities to respond.

2. **Active Student Response.** The student answers the item, solves the problem presented, or completes the academic task. Answering the teacher’s question, computing the answer to a math word problem (and showing all work), and typing in the correct spelling of an item when playing an educational computer game are all examples of active student responding.

3. **Performance Feedback.** The student receives timely feedback about whether his or her response is correct—often with praise and encouragement. A teacher exclaiming ‘Right! Good job!’ when a student gives an response in class, a student using an answer key to check her answer to a math word problem, and a computer message that says ‘Congratulations! You get 2 points for correctly spelling this word!’ are all examples of performance feedback.

Big Ideas: The Four Stages of Learning Can Be Summed Up in the ‘Instructional Hierarchy’
(Haring et al., 1978)

Student learning can be thought of as a multi-stage process. The universal stages of learning include:

- **Acquisition**: The student is just acquiring the skill.
- **Fluency**: The student can perform the skill but must make that skill ‘automatic’.
- **Generalization**: The student must perform the skill across situations or settings.
- **Adaptation**: The student confronts novel task demands that require that the student adapt a current skill to meet new requirements.

Rehearsal of ‘Math Facts’

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 5 = ___</td>
<td>2 x 6 = ___</td>
<td>5 x 5 = ___</td>
</tr>
<tr>
<td>3 x 2 = ___</td>
<td>3 x 8 = ___</td>
<td>5 x 3 = ___</td>
</tr>
<tr>
<td>6 x 5 = ___</td>
<td>9 x 2 = ___</td>
<td>3 x 6 = ___</td>
</tr>
<tr>
<td>8 x 2 = ___</td>
<td>4 x 7 = ___</td>
<td>8 x 4 = ___</td>
</tr>
<tr>
<td>9 x 7 = ___</td>
<td>7 x 6 = ___</td>
<td>3 x 5 = ___</td>
</tr>
</tbody>
</table>

Step 1: The tutor writes down on a series of index cards the math facts that the student needs to learn. The problems are written without the answers.
Step 2: The tutor reviews the ‘math fact’ cards with the student. Any card that the student can answer within 2 seconds is sorted into the ‘KNOWN’ pile. Any card that the student cannot answer within two seconds—or answers incorrectly—is sorted into the ‘UNKNOWN’ pile.

<table>
<thead>
<tr>
<th>‘KNOWN’ Facts</th>
<th>‘UNKNOWN’ Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 5 = ___</td>
<td>3 x 8 = ___</td>
</tr>
<tr>
<td>2 x 6 = ___</td>
<td>3 x 8 = ___</td>
</tr>
<tr>
<td>3 x 2 = ___</td>
<td>5 x 3 = ___</td>
</tr>
<tr>
<td>5 x 3 = ___</td>
<td>9 x 2 = ___</td>
</tr>
<tr>
<td>3 x 6 = ___</td>
<td>8 x 4 = ___</td>
</tr>
<tr>
<td>8 x 4 = ___</td>
<td>5 x 5 = ___</td>
</tr>
<tr>
<td>6 x 5 = ___</td>
<td>8 x 2 = ___</td>
</tr>
<tr>
<td>4 x 7 = ___</td>
<td>8 x 2 = ___</td>
</tr>
<tr>
<td>9 x 7 = ___</td>
<td>3 x 5 = ___</td>
</tr>
<tr>
<td>7 x 6 = ___</td>
<td></td>
</tr>
</tbody>
</table>
Math Review: Incremental Rehearsal of ‘Math Facts’

Step 3: The tutor begins to follow a nine-step incremental-rehearsal procedure with the student. First, the tutor presents the student with a single index card containing an ‘unknown’ math fact. The tutor reads the problem aloud, gives the student a series of ‘known’ math facts, and asks the student to answer the whole series of math facts—until the student gives the correct answer, then prompts the student to read off the same unknown problem and provide the correct answer.

The review deck contains a total of one ‘unknown’ math fact and nine ‘known’ math facts (a ratio of 90 percent ‘known’ to 10 percent ‘unknown’ material).

3 x 8 = __  4 x 5 = __  2 x 6 = __
3 x 2 = __  3 x 6 = __  5 x 3 = __
8 x 4 = __  6 x 5 = __  4 x 7 = __
Math Review: Incremental Rehearsal of ‘Math Facts’

Step 4: A new student is presented with facts that have not been mastered by the student yet, starting with sets of 'known' math facts interspersed with a single 'unknown' math fact to answer—and the review sequence is once again repeated each time until the student answers the ‘unknown’ math fact correctly three times. Daily review sessions are discontinued when time runs out or when the student answers an ‘unknown’ math fact incorrectly three times.

\[
\begin{align*}
9 \times 2 &= __ \\
3 \times 8 &= __ \\
4 \times 5 &= __ \\
2 \times 0 &= __ \\
3 \times 0 &= __ \\
3 \times 6 &= __ \\
8 \times 3 &= __ \\
6 \times 5 &= __ \\
\emptyset \times 5 &= __
\end{align*}
\]
Acquisition Stage: Cover-Copy-Compare:
Math Computational Fluency-Building Intervention

The student is given sheet with correctly completed math problems in left column and index card.

For each problem, the student:
- studies the model
- covers the model with index card
- copies the problem from memory
- solves the problem
- uncovers the correctly completed model to check answer

Acquisition Stage: Math Computation: Motivate With ‘Errorless Learning’ Worksheets

In this version of an ‘errorless learning’ approach, the student is directed to complete math facts as quickly as possible. If the student comes to a number problem that he or she cannot solve, the student is encouraged to locate the problem and its correct answer in the key at the top of the page and write it in. This idea works best for basic math facts.

Such speed drills build computational fluency while promoting students’ ability to visualize and to use a mental number line.

TIP: Consider turning this activity into a ‘speed drill’. The student is given a kitchen timer and instructed to set the timer for a predetermined span of time (e.g., 2 minutes) for each drill. The student completes as many problems as possible before the timer rings. The student then graphs the number of problems correctly computed each day on a time-series graph, attempting to better his or her previous score.

Source: Caron, T. A. (2007). Learning multiplication the easy way. The Clearing House, 80, 278-282
## ‘Errorless Learning’ Worksheet Sample

**Curriculum-Based Assessment Mathematics**  
**Multiple-Skills Computation Probe: Examiner Copy**

<table>
<thead>
<tr>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2 CD/2 CD Total</strong></td>
<td><strong>2 CD/4 CD Total</strong></td>
<td><strong>2 CD/6 CD Total</strong></td>
</tr>
<tr>
<td><strong>SUBTRACTION:</strong> 1-digit number from a 1- or 2-digit number: no regrouping</td>
<td><strong>MULTIPLICATION:</strong> 2-digit number times 1-digit number: no regrouping</td>
<td><strong>SUBTRACTION:</strong> 1-digit number from a 1- or 2-digit number: no regrouping</td>
</tr>
<tr>
<td>26</td>
<td>34</td>
<td>38</td>
</tr>
<tr>
<td>(-) 11</td>
<td>(\times) 2</td>
<td>(-) 22</td>
</tr>
<tr>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>15</td>
<td>68</td>
<td>16</td>
</tr>
</tbody>
</table>

Source: Caron, T. A. (2007). Learning multiplication the easy way. The Clearing House, 80, 278-282
1. The student is given a math computation worksheet of a specific problem type, along with an answer key [Academic Opportunity to Respond].

2. The student consults his or her performance chart and notes previous performance. The student is encouraged to try to ‘beat’ his or her most recent score.

3. The student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems as possible. The student sets a timer and works on the computation sheet until the timer rings. [Active Student Responding]

4. The student checks his or her work, giving credit for each correct digit (digit of correct value appearing in the correct place-position in the answer). [Performance Feedback]

5. The student records the day’s score of TOTAL number of correct digits on his or her personal performance chart.

6. The student receives praise or a reward if he or she exceeds the most recently posted number of correct digits.

Self-Administered Arithmetic Combination Drills:
Examples of Student Worksheet and Answer Key

Worksheets created using Math Worksheet Generator. Available online at:
http://www.interventioncentral.org/htmdocs/tools/mathprobe/addsing.php
Response to Intervention

Self-Administered Arithmetic Combination Drills...

Johnny’s Multiplication-Facts Chart

Math Facts: Multiplication

Reward Given

Reward Given

Reward Given

No Reward

No Reward

No Reward

Correct Digits Per Five Minutes

0 5 10 15 20 25 30 35 40 45

05Mar06 08Mar06 12Mar06 15Mar06

Instructional Days

www.interventioncentral.org
How to... Use PPT Group Timers in the Classroom

Response to Intervention

Intervention Central offers free tools and resources to help school staff and parents promote positive classroom behaviors and foster effective learning for all children and youth. The site was created by Jim Wright, a school psychologist and school administrator from Central New York.

Visit to check out newly posted academic and behavioral intervention strategies, download publications on effective teaching practices, and use tools that streamline classroom assessment and intervention.

Internet Course [PAY]

Jim Wright has created a modestly priced on-line course on Positive Behavior Strategies. Developed in collaboration with the company Professional Learning Board, this course packs the best classroom and individual behavior management ideas from Intervention Central into a convenient, time-efficient class. Read a description of the course or check out a brief course preview.

On-Line Tools

- Behavior Reporter, Behavior Report Card Generator
- Curriculum-Based Measurement List Builder
- Jackpot! On-line Reinforcer Survey Generator
- Math Worksheet Generator
- OKAPI Reading Probe Generator
- ChartDog: Create CBM Charts
- Test Score Analyzer 2.0

Favorite Downloads

- ADI ID Evaluation Manual
- Bully Prevention Booklet
- Curriculum-Based Measurement Warehouse
- Peer Tutor Training Manual
- Reading Interventions Manual

Response to Intervention Central: Your Site for Response To Intervention Resources - Windows Internet Explorer provided by Yahoo!
Motivation: Math Computation: Chunking

- Break longer assignments into shorter assignments with performance feedback given after each shorter ‘chunk’ (e.g., break a 20-minute math computation worksheet task into 3 seven-minute assignments). Breaking longer assignments into briefer segments also allows the teacher to praise struggling students more frequently for work completion and effort, providing an additional ‘natural’ reinforcer.

Motivation: Math Computation: Problem Interspersal Technique p. 22

- The teacher first identifies the range of ‘challenging’ problem-types (number problems appropriately matched to the student’s current instructional level) that are to appear on the worksheet.

- Then the teacher creates a series of ‘easy’ problems that the students can complete very quickly (e.g., adding or subtracting two 1-digit numbers). The teacher next prepares a series of student math computation worksheets with ‘easy’ computation problems interspersed at a fixed rate among the ‘challenging’ problems.

- If the student is expected to complete the worksheet independently, ‘challenging’ and ‘easy’ problems should be interspersed at a 1:1 ratio (that is, every ‘challenging’ problem in the worksheet is preceded and/or followed by an ‘easy’ problem).

- If the student is to have the problems read aloud and then asked to solve the problems mentally and write down only the answer, the items should appear on the worksheet at a ratio of 3 ‘challenging’ problems for every ‘easy’ one (that is, every 3 ‘challenging’ problems are preceded and/or followed by an ‘easy’ one).

Team Activity: ‘Universal’ Math Ideas

• Review the brief math intervention/accommodation ideas that appear on pp. 14-19 in your handout.

• Select at least one idea that you would like all teachers in your department or school to consider using.
Building Student Skills in Applied Math Problems

Jim Wright
www.interventioncentral.org
‘Advanced Math’ Quotes from Yogi Berra—

- “Ninety percent of the game is half mental.”
- “Pair up in threes.”
- “You give 100 percent in the first half of the game, and if that isn't enough in the second half you give what's left.”
Defining Goals & Challenges in Applied Math
Potential ‘Blockers’ of Higher-Level Math Problem-Solving: A Sampler

- Limited reading skills
- Failure to master—or develop automaticity in—basic math operations
- Lack of knowledge of specialized math vocabulary (e.g., ‘quotient’)
- Lack of familiarity with the specialized use of known words (e.g., ‘product’)
- Inability to interpret specialized math symbols (e.g., ‘4 < 2’)
- Difficulty ‘extracting’ underlying math operations from word/story problems
- Difficulty identifying and ignoring extraneous information included in word/story problems
Interpreting Math Graphics: A Reading Comprehension Intervention

www.interventioncentral.org
Housing Bubble

Graphic: New York Times

23 September 2007

Housing Price Index = 171 in 2005

2007

Housing Price Index = 100 in 1987

U.S. Housing Prices Since 1987: This index is based on sales prices of standard existing single-family homes (not new construction). It has been adjusted for inflation.

The 1987 benchmark is 100 on the chart. If a standard house sold in 1987 for $100,000 (inflation-adjusted to today's dollars), an equivalent house would have sold for $82,000 at the end of 1990 (82 on the index scale).

The index peaked at 171 at the end of 2005, when the same house would have sold for $171,000, a gain of 71 percent.

Sources: Standard & Poor's/Case-Shiller Home Price Index; Bureau of Labor Statistics.

As Prices Soared, Warnings of a Bust...

May 2003: The Economist magazine publishes a survey on global property prices. "Another Bubble Fit to Burst."

May 2004: The economist and real estate skeptic Dean Baker sells his two-bedroom condo in the Adams Morgan neighborhood in Washington because he believes the gains in home prices are unsustainable.


May 2005: Alan Greenspan says: "Without calling the overall national issue a bubble, it's pretty clear that it's an unsustainable underlying pattern."


Feb. 2006: Ben S. Bernanke, the Federal Reserve chairman, says policy makers "expect the housing market to cool but not to change very sharply."

www.interventioncentral.org 61
Response to Intervention

Classroom Challenges in Interpreting Math Graphics

When encountering math graphics, students may:

- expect the answer to be easily accessible when in fact the graphic may expect the reader to interpret and draw conclusions
- be inattentive to details of the graphic
- treat irrelevant data as ‘relevant’
- not pay close attention to questions before turning to graphics to find the answer
- fail to use their prior knowledge both to extend the information on the graphic and to act as a possible ‘check’ on the information that it presents.

Response to Intervention

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics

Students can be more savvy interpreters of graphics in applied math problems by applying the Question-Answer Relationship (QAR) strategy. Four Kinds of QAR Questions:

- RIGHT THERE questions are fact-based and can be found in a single sentence, often accompanied by 'clue' words that also appear in the question.
- THINK AND SEARCH questions can be answered by information in the text but require the scanning of text and making connections between different pieces of factual information.
- AUTHOR AND YOU questions require that students take information or opinions that appear in the text and combine them with the reader's own experiences or opinions to formulate an answer.
- ON MY OWN questions are based on the students' own experiences and do not require knowledge of the text to answer.


www.interventioncentral.org
Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

1. DISTINGUISHING DIFFERENT KINDS OF GRAPHICS. Students are taught to differentiate between common types of graphics: e.g., table (grid with information contained in cells), chart (boxes with possible connecting lines or arrows), picture (figure with labels), line graph, bar graph.

Students note significant differences between the various graphics, while the teacher records those observations on a wall chart. Next students are given examples of graphics and asked to identify which general kind of graphic each is.

Finally, students are assigned to go on a ‘graphics hunt’, locating graphics in magazines and newspapers, labeling them, and bringing to class to review.

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

2. INTERPRETING INFORMATION IN GRAPHICS. Students are paired off, with stronger students matched with less strong ones. The teacher spends at least one session presenting students with examples from each of the graphics categories.

The presentation sequence is ordered so that students begin with examples of the most concrete graphics and move toward the more abstract: Pictures > tables > bar graphs > charts > line graphs.

At each session, student pairs examine graphics and discuss questions such as: “What information does this graphic present? What are strengths of this graphic for presenting data? What are possible weaknesses?”

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

3. LINKING THE USE OF QARS TO GRAPHICS. Students are given a series of data questions and correct answers, with each question accompanied by a graphic that contains information needed to formulate the answer.

Students are also each given index cards with titles and descriptions of each of the 4 QAR questions: RIGHT THERE, THINK AND SEARCH, AUTHOR AND YOU, ON MY OWN.

Working in small groups and then individually, students read the questions, study the matching graphics, and ‘verify’ the answers as correct. They then identify the type question being asked using their QAR index cards.

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

4. USING QARS WITH GRAPHICS INDEPENDENTLY. When students are ready to use the QAR strategy independently to read graphics, they are given a laminated card as a reference with 6 steps to follow:

A. Read the question,
B. Review the graphic,
C. Reread the question,
D. Choose a QAR,
E. Answer the question, and
F. Locate the answer derived from the graphic in the answer choices offered.

Students are strongly encouraged NOT to read the answer choices offered until they have first derived their own answer, so that those choices don’t short-circuit their inquiry.

Developing Student Metacognitive Abilities p. 29
Importance of Metacognitive Strategy Use...

“Metacognitive processes focus on self-awareness of cognitive knowledge that is presumed to be necessary for effective problem solving, and they direct and regulate cognitive processes and strategies during problem solving...That is, successful problem solvers, consciously or unconsciously (depending on task demands), use self-instruction, self-questioning, and self-monitoring to gain access to strategic knowledge, guide execution of strategies, and regulate use of strategies and problem-solving performance.” p. 231

Elements of Metacognitive Processes

“Self-instruction helps students to identify and direct the problem-solving strategies prior to execution. Self-questioning promotes internal dialogue for systematically analyzing problem information and regulating execution of cognitive strategies. Self-monitoring promotes appropriate use of specific strategies and encourages students to monitor general performance. [Emphasis added].”

Combining Cognitive & Metacognitive Strategies to Assist Students With Mathematical Problem Solving

Solving an advanced math problem independently requires the coordination of a number of complex skills. The following strategies combine both cognitive and metacognitive elements (Montague, 1992; Montague & Dietz, 2009). First, the student is taught a 7-step process for attacking a math word problem (cognitive strategy). Second, the instructor trains the student to use a three-part self-coaching routine for each of the seven problem-solving steps (metacognitive strategy).
Cognitive Portion of Combined Problem Solving Approach

In the cognitive part of this multi-strategy intervention, the student learns an explicit series of steps to analyze and solve a math problem. Those steps include:

1. **Reading the problem.** The student reads the problem carefully, noting and attempting to clear up any areas of uncertainly or confusion (e.g., unknown vocabulary terms).
2. **Paraphrasing the problem.** The student restates the problem in his or her own words.
3. **‘Drawing’ the problem.** The student creates a drawing of the problem, creating a visual representation of the word problem.
4. **Creating a plan to solve the problem.** The student decides on the best way to solve the problem and develops a plan to do so.
5. **Predicting/Estimating the answer.** The student estimates or predicts what the answer to the problem will be. The student may compute a quick approximation of the answer, using rounding or other shortcuts.
6. **Computing the answer.** The student follows the plan developed earlier to compute the answer to the problem.
7. **Checking the answer.** The student methodically checks the calculations for each step of the problem. The student also compares the actual answer to the estimated answer calculated in a previous step to ensure that there is general agreement between the two values.
Metacognitive Portion of Combined Problem Solving Approach

The metacognitive component of the intervention is a three-part routine that follows a sequence of ‘Say’, ‘Ask, ‘Check’. For each of the 7 problem-solving steps reviewed above:

- The student first self-instructs by stating, or ‘saying’, the purpose of the step (‘Say’).
- The student next self-questions by ‘asking’ what he or she intends to do to complete the step (‘Ask’).
- The student concludes the step by self-monitoring, or ‘checking’, the successful completion of the step (‘Check’).
# Combined Cognitive & Metacognitive Elements of Strategy

Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
</tr>
</thead>
</table>
| 1. **Read the problem.** | ‘Say’ (Self-Instruction) Target: *The student reads and studies the problem carefully before proceeding.*  
‘Ask’ (Self-Question) Target: *Does the student fully understand the problem?*  
‘Check’ (Self-Monitor) Target: *Proceed only if the problem is understood.* | Say: “I will read the problem. I will reread the problem if I don’t understand it.”  
Ask: “Now that I have read the problem, do I fully understand it?”  
Check: “I understand the problem and will move forward.” |
# Combined Cognitive & Metacognitive Elements of Strategy

## Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. <strong>Paraphrase the problem.</strong></td>
<td>‘Say’ (Self-Instruction) Target: <em>The student restates the problem in order to demonstrate understanding.</em> &lt;br&gt;‘Ask’ (Self-Question) Target: <em>Is the student able to paraphrase the problem?</em> &lt;br&gt;‘Check’ (Self-Monitor) Target: <em>Ensure that any highlighted key words are relevant to the question.</em></td>
<td><strong>Say:</strong> “I will highlight key words and phrases that relate to the problem question.”&lt;br&gt;“I will restate the problem in my own words.”&lt;br&gt;<strong>Ask:</strong> “Did I highlight the most important words or phrases in the problem?”&lt;br&gt;<strong>Check:</strong> “I found the key words or phrases that will help to solve the problem.”</td>
</tr>
</tbody>
</table>
## Combined Cognitive & Metacognitive Elements of Strategy

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
</tr>
</thead>
</table>
| 3. ‘Draw’ the problem.  | ‘Say’ (Self-Instruction) Target: The student creates a drawing of the problem to consolidate understanding.  
‘Ask’ (Self-Question) Target: Is there a match between the drawing and the problem?  
‘Check’ (Self-Monitor) Target: The drawing includes in visual form the key elements of the math problem. | Say: “I will draw a diagram of the problem.”  
Ask: “Does my drawing represent the problem?”  
Check: “The drawing contains the essential parts of the problem.” |

Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)
## Combined Cognitive & Metacognitive Elements of Strategy

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
</tr>
</thead>
</table>
| 4. Create a plan to solve the problem. | ‘Say’ (Self-Instruction) Target: The student generates a plan to solve the problem.  
‘Ask’ (Self-Question) Target: What plan will help the student to solve this problem?  
‘Check’ (Self-Monitor) Target: The plan is appropriate to solve the problem. | Say: “I will make a plan to solve the problem.”  
Ask: “What is the first step of this plan? What is the next step of the plan?”  
Check: “My plan has the right steps to solve the problem.” |
### Combined Cognitive & Metacognitive Elements of Strategy

**Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)**

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
</tr>
</thead>
</table>
| 5. **Predict/estimate the Answer.** | ‘Say’ (Self-Instruction) **Target**: The student uses estimation or other strategies to predict or estimate the answer.  
‘Ask’ (Self-Question) **Target**: What estimating technique will the student use to predict the answer?  
‘Check’ (Self-Monitor) **Target**: The predicted/estimated answer used all of the essential problem information. | **Say**: “I will estimate what the answer will be.”  
**Ask**: “What numbers in the problem should be used in my estimation?”  
**Check**: “I did not skip any important information in my estimation.” |
# Combined Cognitive & Metacognitive Elements of Strategy

Table 1: ‘Say-Aask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Aask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Aask-Check’ Prompts</th>
</tr>
</thead>
</table>
| 6. **Compute the answer.** | ‘Say’ (Self-Instruction) Target: The student follows the plan to compute the solution to the problem.  
‘Ask’ (Self-Question) Target: Does the answer agree with the estimate?  
‘Check’ (Self-Monitor) Target: The steps in the plan were followed and the operations completed in the correct order. | Say: “I will compute the answer to the problem.”  
Ask: “Does my answer sound right?” “Is my answer close to my estimate?”  
Check: “I carried out all of the operations in the correct order to solve this problem.” |
## Combined Cognitive & Metacognitive Elements of Strategy

Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
</tr>
</thead>
</table>
| 7. **Check the answer.**| ‘Say’ (**Self-Instruction**) **Target**: The student reviews the computation steps to verify the answer.  
‘Ask’ (**Self-Question**) **Target**: Did the student check all the steps in solving the problem and are all computations correct?  
‘Check’ (**Self-Monitor**) **Target**: The problem solution appears to have been done correctly. | Say: “I will check the steps of my answer.”  
Ask: “Did I go through each step in my answer and check my work?”  
Check: “” |
Applied Problems: Pop Quiz

7-Step Problem-Solving: Process

1. Reading the problem.
2. Paraphrasing the problem.
3. ‘Drawing’ the problem.
4. Creating a plan to solve the problem.
5. Predicting/Estimating the answer.
6. Computing the answer.
7. Checking the answer.

Directions: As a team, read the following problem. At your tables, apply the 7-step problem-solving (cognitive) strategy to complete the problem. As you complete each step of the problem, apply the ‘Say, Ask, Check’ metacognitive sequence. Try to complete the entire 7 steps within the time allocated for this exercise.

Q: “To move their armies, the Romans built over 50,000 miles of roads. Imagine driving all those miles! Now imagine driving those miles in the first gasoline-driven car that has only three wheels and could reach a top speed of about 10 miles per hour. For safety’s sake, let’s bring along a spare tire. As you drive the 50,000 miles, you rotate the spare with the other tires so that all four tires get the same amount of wear. Can you figure out how many miles of wear each tire accumulates?”

A: “Since the four wheels of the three-wheeled car share the journey equally, simply take three-fourths of the total distance (50,000 miles) and you’ll get 37,500 miles for each tire.”

Team Activity: Cognitive/Metacognitive Strategy

- Review the cognitive/meta-cognitive strategies shared in this workshop (Montague, 1992).

- Discuss how you might use/adapt this framework for use in your classroom or school.
Challenge: Measuring Student Progress on Math Interventions
Educational Decisions and Corresponding Types of Assessment

- SCREENING/BENCHMARKING DECISIONS: Tier 1: Brief screenings to quickly indicate whether students in the general-education population are academically proficient or at risk.

- PROGRESS-MONITORING DECISIONS: At Tiers 1, 2, and 3, ongoing ‘formative’ assessments to judge whether students on intervention are making adequate progress.

- INSTRUCTIONAL/DIAGNOSTIC DECISIONS: At any Tier, detailed assessment to map out specific academic deficits, discover the root cause(s) of a student’s academic problem.

- OUTCOME DECISIONS: Summative assessment (e.g., state tests) to evaluate the effectiveness of a program.

Response to Intervention

RTI Screening and Progress-Monitoring

To measure student ‘response to instruction/intervention’ effectively, the RTI model measures students’ academic performance and progress on schedules matched to each student’s risk profile and intervention Tier membership.

- **Benchmarking/Universal Screening.** All children in a grade level are assessed at least 3 times per year on a common collection of math and/or other academic assessments.

- **Strategic Monitoring.** Students placed in Tier 2 (supplemental) intervention groups are assessed 2 times per month to gauge their progress with this intervention.

- **Intensive Monitoring.** Students who participate in an intensive, individualized Tier 3 intervention are assessed at least once per week.


www.interventioncentral.org
Response to Intervention

Curriculum-Based Measurement: Advantages as a Set of Tools to Monitor RTI/Academic Cases

- **Aligns** with curriculum-goals and materials
- Is **reliable** and **valid** (has ‘technical adequacy’)
- Is **criterion-referenced**: sets specific performance levels for specific tasks
- Uses **standard procedures** to prepare materials, administer, and score
- Samples student performance to give objective, observable ‘low-inference’ information about student performance
- Has **decision rules** to help educators to interpret student data and make appropriate instructional decisions
- Is **efficient** to implement in schools (e.g., training can be done quickly; the measures are brief and feasible for classrooms, etc.)
- Provides data that can be converted into **visual displays** for ease of communication


www.interventioncentral.org
Response to Intervention

Clearinghouse for RTI Screening and Progress-Monitoring Tools

- The National Center on RTI (www.rti4success.org) maintains pages rating the technical adequacy of RTI screening and progress-monitoring tools.

<table>
<thead>
<tr>
<th>Tools</th>
<th>Area</th>
<th>Reliability of the Performance Level Score</th>
<th>Reliability of the Slope</th>
<th>Validity of the Performance Level Score</th>
<th>Predictive Validity of the Slope of Improvement</th>
<th>Alternate Forms</th>
<th>Sensitive to Student Improvement</th>
<th>End-of-Year Benchmarks</th>
<th>Rates of Improvement Specified</th>
<th>Norms Disagg. for Diverse Populations</th>
<th>Disagg. Regulated Reliability and Validity Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMSweb</td>
<td>Math</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>No</td>
<td>●</td>
</tr>
<tr>
<td>AIMSweb</td>
<td>Oral Reading</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>No</td>
<td>●</td>
</tr>
<tr>
<td>AIMSweb</td>
<td>Test of Early Literacy - Letter Naming Fluency</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>No</td>
<td>●</td>
</tr>
<tr>
<td>AIMSweb</td>
<td>Test of Early Literacy - Letter Sound Fluency</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>No</td>
<td>●</td>
</tr>
</tbody>
</table>

www.interventioncentral.org
CBM: Math Computation
CBM Math Computation Probes:
Preparation
CBM Math Computation Sample Goals

- Addition: Add two one-digit numbers: sums to 18
- Addition: Add 3-digit to 3-digit with regrouping from ones column only
- Subtraction: Subtract 1-digit from 2-digit with no regrouping
- Subtraction: Subtract 2-digit from 3-digit with regrouping from ones and tens columns
- Multiplication: Multiply 2-digit by 2-digit-no regrouping
- Multiplication: Multiply 2-digit by 2-digit with regrouping
CBM Math Computation
Assessment: Preparation

- Select either single-skill or multiple-skill math probe format.
- Create student math computation worksheet (including enough problems to keep most students busy for 2 minutes)
- Create answer key
CBM Math Computation
Assessment: Preparation

• Advantage of single-skill probes:
  – Can yield a more ‘pure’ measure of student’s computational fluency on a particular problem type
CBM Math Computation
Assessment: Preparation

• Advantage of multiple-skill probes:
  – Allow examiner to gauge student’s adaptability between problem types (e.g., distinguishing operation signs for addition, multiplication problems)
  – Useful for including previously learned computation problems to ensure that students retain knowledge.
Response to Intervention

Curriculum-Based Assessment Mathematics
Multiple-Skills Computation Probe: Examiner Copy

Item 1:
4 CD/4 CD Total
ADDITION: Three to five 3-digit numbers: Regrouping in any column

663
+208
+628
+411
1910

Item 2:
16 CD/20 CD Total
DIVISION: 4-digit number divided by 2-digit number: no remainder

23/4439
-23
213
-207
69
-69
0

Item 3:
14 CD/34 CD Total
MULTIPLICATION: 4-digit number times 2-digit number: no regrouping

2213
x 12
4426
2213-
26,556

Item 4:
5 CD/39 CD Total
SUBTRACTION: 5-digit number from a 5-digit number: regrouping from 1's and 10's columns

36,841
- 15,765
21,076

Item 5:
4 CD/43 CD Total
ADDITION: Three to five 3-digit numbers: Regrouping in any column

290
+731
+672
1693

Item 6:
12 CD/55 CD Total
DIVISION: 4-digit number divided by 2-digit number: no remainder

68/1496
-136
136
-136
0

www.interventioncentral.org
CBM Math Computation Probes:
Administration
Administration of CBM math probes

The examiner distributes copies of one or more math probes to all the students in the group. (Note: These probes may also be administered individually). The examiner says to the students:

The sheets on your desk are math facts.

If the students are to complete a single-skill probe, the examiner then says: All the problems are [addition or subtraction or multiplication or division] facts.

If the students are to complete a multiple-skill probe, the examiner then says: There are several types of problems on the sheet. Some are addition, some are subtraction, some are multiplication, and some are division [as appropriate]. Look at each problem carefully before you answer it.

When I say 'start,' turn them over and begin answering the problems. Start on the first problem on the left on the top row [point]. Work across and then go to the next row. If you can't answer the problem, make an 'X' on it and go to the next one. If you finish one side, go to the back. Are there any questions?

Say, Start. The examiner starts the stopwatch.

While the students are completing worksheets, the examiner and any other adults assisting in the assessment circulate around the room to ensure that students are working on the correct sheet, that they are completing problems in the correct order (rather than picking out only the easy items), and that they have pencils, etc.

After 2 minutes have passed, the examiner says Stop. CBM math probes are collected for scoring.
CBM Math Computation Probes: Scoring
CBM Math Computation
Assessment: Scoring

Unlike more traditional methods for scoring math computation problems, CBM gives the student credit for each correct digit in the answer. This approach to scoring is more sensitive to short-term student gains and acknowledges the child’s partial competencies in math.
Math Computation: Scoring Examples

\[ \frac{56}{5880} \]

\[ -56 \]

\[ -280 \]

\[ -280 \]

12 CDs
Math Computation: Scoring

Numbers Above Line Are Not Counted
Placeholders Are Counted

48
x 32
96
1440
1536
### CBM Math Computation Norms (Fuchs, Fuchs, Hintze, & Lembke, 2007)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Assessment Time</th>
<th>End of Year Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergarten</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Grade 1</td>
<td>2 minutes</td>
<td>20 digits</td>
</tr>
<tr>
<td>Grade 2</td>
<td>2 minutes</td>
<td>20 digits</td>
</tr>
<tr>
<td>Grade 3</td>
<td>3 minutes</td>
<td>30 digits</td>
</tr>
<tr>
<td>Grade 4</td>
<td>3 minutes</td>
<td>40 digits</td>
</tr>
<tr>
<td>Grade 5</td>
<td>5 minutes</td>
<td>30 digits</td>
</tr>
<tr>
<td>Grade 6</td>
<td>6 minutes</td>
<td>35 digits</td>
</tr>
</tbody>
</table>

Question: How can a school use CBM Math Computation probes if students are encouraged to use one of several methods to solve a computation problem—and have no fixed algorithm?

Answer: Students should know their ‘math facts’ automatically. Therefore, students can be given math computation probes to assess the speed and fluency of basic math facts—even if their curriculum encourages a variety of methods for solving math computation problems.
RTI Mathematics Screening Tools

- **Math Concepts & Applications.** The Math Concepts and Applications assessments (www.easycbm.com) were developed using as a guide the Math Curriculum Focal Points from the National Council of Teachers of Mathematics (NCTM).
The application to create CBM Early Math Fluency probes online

Examples of Early Math Fluency
(Number Sense) CBM Probes

Quantity Discrimination

| 4 | 12 |

Missing Number

| 14 |   | 16 | 17 |

Number Identification

<table>
<thead>
<tr>
<th>34</th>
<th>37</th>
<th>50</th>
<th>38</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>24</td>
<td>35</td>
<td>15</td>
<td>40</td>
</tr>
</tbody>
</table>


**Quantity Discrimination (QD)**

**Description:** The student is given a sheet of number pairs and must verbally identify the larger of the two values for each pair.

Select the **lowest** and **highest** numbers to be selected in the quantity-discrimination items:

FROM 0 TO 20

How many quantity discrimination items should appear **in each row**?

3 items

How many rows of items should appear on the student worksheet?

8 rows

Submit

**QD Directions:** Download directions for administering and scoring **Quantity Discrimination** probes, test statistics, & brief guidelines for use in an RTI process

**QD Graph:** Access a time-series graph to chart student progress using **Quantity Discrimination** probes

---

**Missing Number (MN)**

**Description:** The student is given a sheet that contains a series of 3- or 4-number sequences. In each sequence, one number is missing.
## RTI Mathematics Screening Tools

**Math Concepts & Applications.** The Math Concepts and Applications assessments (www.easycbm.com) were developed using as a guide the Math Curriculum Focal Points from the National Council of Teachers of Mathematics (NCTM).

### Math Numbers and Operations 3_3

**Student Name:** ____________________________  
**Date:** ____________________________

1.  
Which is a hexagon?

- A. A  
- B. B  
- C. C  

2.  
How many parts in all?

- A. 6  
- B. 7  
- C. 3

3.  
Rico takes all 3 balls.  
How much did he take?

4.  
gray part = ___
Team Activity: Using Formative Math Assessments to Inform Instruction/Intervention

• Review the math measures just discussed for screening and monitoring the progress of students in math skills.

• Have a discussion about how you or others in your school might explore/pilot/use these math assessment tools.
Elementary Tier 1
Intervention: Case Example:
John: Math Computation

Jim Wright
www.interventioncentral.org
Case Example: Math Computation

The Problem

• John is a fourth-grade student. His teacher, Mrs. Kennedy, is concerned that John appears to be much slower in completing math computation items than are his classmates.
Profile of Students With Significant Math Difficulties

1. **Spatial organization.** The student commits errors such as misaligning numbers in columns in a multiplication problem or confusing directionality in a subtraction problem (and subtracting the original number—minuend—from the figure to be subtracted (subtrahend).

2. **Visual detail.** The student misreads a mathematical sign or leaves out a decimal or dollar sign in the answer.

3. **Procedural errors.** The student skips or adds a step in a computation sequence. Or the student misapplies a learned rule from one arithmetic procedure when completing another, different arithmetic procedure.

4. **Inability to ‘shift psychological set’**. The student does not shift from one operation type (e.g., addition) to another (e.g., multiplication) when warranted.

5. **Graphomotor.** The student’s poor handwriting can cause him or her to misread handwritten numbers, leading to errors in computation.

6. **Memory.** The student fails to remember a specific math fact needed to solve a problem. (The student may KNOW the math fact but not be able to recall it at ‘point of performance’.)

7. **Judgment and reasoning.** The student comes up with solutions to problems that are clearly unreasonable. However, the student is not able adequately to evaluate those responses to gauge whether they actually make sense in context.

Case Example: Math Computation

Core Instruction

• John’s school uses the *Everyday Math* curriculum (McGraw Hill/University of Chicago). In addition to the basic curriculum the series contains intervention exercises for students who need additional practice or remediation.

As an extension of core instruction, his teacher works with a small group of children in her room—including John—having them complete these practice exercises to boost their math computation fluency. While other children in this group appear to benefit from the assistance, John does not make noticeable gains in his computation speed.
Case Example: Math Computation

The Evidence

• Mrs. Kennedy collects and reviews information that may be relevant to understanding John’s math computation concern:

  Teacher Interview. Ms. Kennedy talks with John’s Grade 3 teacher from last year who confirms that John was slow in completing math facts in that setting as well—but was accurate in his work and appeared motivated to do computation assignments.
Case Example: Math Computation

The Evidence (Cont.)

- **Review of Records.** When Mrs. Kennedy reviews John’s past report cards and other records from his cumulative file, she does not find any comments or other evidence that he displayed fine-motor delays that might interfere with computation fluency.

- **Work Products.** Mrs. Kennedy reviews examples of John’s work on untimed math computation worksheets. Similar to observations shared by the 3rd grade teacher, Mrs. Kennedy notes that John’s work is accurate—even though he did not complete as many problems as peers.
Case Example: Math Computation

The Evidence (Cont.)

- *Direct Observation*. Watching John complete a computation worksheet, his teacher notes that John counts on his fingers. This appears to slow down his computation speed considerably.
Response to Intervention

Case Example: Math Computation

The Intervention

• Mrs. Kennedy met with a consultant to create a Tier 1 (classroom) intervention plan for John. Both the consultant and teacher agreed that John was slow in math computation because he relied on finger counting to compute number problems rather than using the more efficient strategies of mental arithmetic and automatic recall of math facts.
Case Example: Math Computation

The Intervention (Cont.)

- Mrs. Kennedy decided to institute a version of math computation time-drills as a technique to boost John’s computation speed and (she hoped) encourage him to give up the finger-counting habit.

Each day, John would self-administer and score 3 separate three-minute time drills using multiplication facts...
Math Intervention: Tier I or II: Elementary & Secondary:

Self-Administered ‘Math Fact’ Timed Drills With Performance Self-Monitoring & Incentives

1. The student is given a math computation worksheet of a specific problem type, along with an answer key [Academic Opportunity to Respond].

2. The student consults his or her performance chart and notes previous performance. The student is encouraged to try to ‘beat’ his or her most recent score.

3. The student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems as possible. The student sets a timer and works on the computation sheet until the timer rings. [Active Student Responding]

4. The student checks his or her work, giving credit for each correct digit (digit of correct value appearing in the correct place-position in the answer). [Performance Feedback]

5. The student records the day’s score of TOTAL number of correct digits on his or her personal performance chart.

6. The student receives praise or a reward if he or she exceeds the most recently posted number of correct digits.

Self-Administered Arithmetic Combination Drills:
Examples of Student Worksheet and Answer Key

Worksheets created using Math Worksheet Generator. Available online at:
http://www.interventioncentral.org/htmdocs/tools/mathprobe/addsing.php
Response to Intervention

Self-Administered Arithmetic Combination Drills...

Johnny's Multiplication-Facts Chart

Math Facts: Multiplication

Reward Given
Reward Given
Reward Given
Reward Given

No Reward
No Reward
No Reward

Correct Digits Per Five Minutes

Instructional Days

05Mar06 08Mar06 12Mar06 15Mar06
Case Example: Math Computation

Documentation and Goal-Setting

• While meeting with the consultant, Mrs. Kennedy filled out a Tier 1 intervention plan for the student. On the plan, she listed interventions to be used, a checkup date (5 instructional weeks), and data to be used to assess student progress.

• Mrs. Kennedy decided to monitor John’s computation progress once per week using a 2-minute curriculum-based measurement math computation probe.
Case Example: Math Computation

Goal-Setting

- Mrs. Kennedy’s school used math computation guidelines that indicated that defined fluency in math computation at 40 correct digits (CDs) or more in two minutes.
- At baseline, John was found to calculate an average of 18 CDs per 2 minutes.
- Mrs. Kennedy decided to set a goal of 2 additional CDs per week. Her intermediate goal was for John to compute at least 28 CDs per 2 minutes at the end of five weeks.

| Curriculum-Based Measurement: Math Computation (Adapted from Deno & Mirkin, 1977) |
|---------------------------------|----------------|----------------|
| Grade  | Digits Correct in 2 Minutes | Digits Incorrect in 2 Minutes |
| 1-3    | 20-38                       | 6-14                        |
| 4 & Up | 40-78                       | 6-14                        |

Comments: These math computation norms are still widely referenced. However, the norms were collected nearly 30 years ago and may not be widely representative because they were drawn from a relatively small sample of students. Additionally, the norms make no distinction between easy and more challenging math computation problem types. Because of these limitations, these norms are best regarded as a rough indicator of ‘typical’ student math computation skills.
### Tier 1/Classroom Intervention Planning Sheet

**Teacher/Team:** Mrs. Kennedy  
**Date:** Oct 10, 2010  
**Student:** John S.

**Student Problem Definition #1:** Slow math computation speed (computes multiplication facts at about half the expected rate)

**Student Problem Definition #2:**

[Optional] Person(s) assisting with intervention planning process: Angela Cordone, Special Education Teacher

#### Interventions: Essential Elements (Witt et al., 2004)
- Clear problem-definition(s)
- Baseline data
- Goal for improvement
- Progress-monitoring plan

<table>
<thead>
<tr>
<th>Intervention Description</th>
<th>Intervention Delivery</th>
<th>Check-Up Date</th>
<th>Assessment Data</th>
</tr>
</thead>
</table>
| Math Computation Timed Drill (see attached description) | Use math worksheet generator on www.interventioncentral.org to create all time-drill and assessment materials. | Five instructional weeks. | Type(s) of Data to Be Used:  
CBM Math Computation: multiplication facts |
|                          |                       |               | Baseline  
18 CDs in 2 mins | Goal by Check-Up  
28 CDs in 2 mins |
Case Example: Math Computation

The Outcome

- When the intervention had been in place for 5 weeks, Mrs. Kennedy found that John had exceeded his intermediate goal of 28 CDs per 2 minutes—the actual number was 34 CDs.
- Mrs. Kennedy judged that the intervention was effective. She decided to continue the intervention without changes for another five weeks with the expectation that John would reach his goal (40 CDs in 2 minutes) by that time.
Case Example: Math Computation

The Outcome (Alternative Ending)

• When the intervention had been in place for 5 weeks, Mrs. Kennedy found that John had not attained his intermediate goal of 28 CDs in 2 minutes. In fact, he had made virtually no progress.

• Mrs. Kennedy decided to try other strategies in the classroom to help John to acquire math facts (e.g., Cover-Copy-Compare).

• However, Mrs. Kennedy also referred the student for additional RTI support (Tier 2 or 3).
Secondary Group-Based Math Intervention Example

- Research indicates that students do well in targeted small-group interventions (4-6 students) when the intervention ‘treatment’ is closely matched to those students’ academic needs (Burns & Gibbons, 2008).

- However, in secondary schools:
  1. students are sometimes grouped for remediation by convenience rather than by presenting need. Teachers instruct across a broad range of student skills, diluting the positive impact of the intervention.
  2. students often present with a unique profile of concerns that does not lend itself to placement in a group intervention.

Response to Intervention

Caution About Secondary Standard-Protocol (‘Group-Based’) Interventions: Avoid the ‘Homework Help’ Trap

- Group-based or standard-protocol interventions are an efficient method for certified teachers to deliver targeted academic support to students (Burns & Gibbons, 2008).

- However, students should be matched to specific research-based interventions that address their specific needs.

- RTI intervention support in secondary schools should not take the form of unfocused ‘homework help’.
Math Mentors: Training Students to Independently Use On-Line Math-Help Resources

1. Math mentors are recruited (school personnel, adult volunteers, student teachers, peer tutors) who have a good working knowledge of algebra.

2. The school meets with each math mentor to verify mentor’s algebra knowledge.

3. The school trains math mentors in 30-minute tutoring protocol, to include:
   A. Requiring that students keep a math journal detailing questions from notes and homework.
   B. Holding the student accountable to bring journal, questions to tutoring session.
   C. Ensuring that a minimum of 25 minutes of 30 minute session are spent on tutoring.

4. Mentors are introduced to online algebra resources (e.g., www.algebrahelp.com, www.math.com) and encouraged to browse them and become familiar with the site content and navigation.
Math Mentors: Training Students to Independently Use On-Line Math-Help Resources

5. Mentors are trained during ‘math mentor’ sessions to:
   A. Examine student math journal
   B. Answer student algebra questions
   C. Direct the student to go online to algebra tutorial websites while mentor supervises. Student is to find the section(s) of the websites that answer their questions.

6. As the student shows increased confidence with algebra and with navigation of the math-help websites, the mentor directs the student to:
   A. Note math homework questions in the math journal
   B. Attempt to find answers independently on math-help websites
   C. Note in the journal any successful or unsuccessful attempts to independently get answers online
   D. Bring journal and remaining questions to next mentoring meeting.
algebra.help
www.algebrahelp.com

Algebrahelp.com is a collection of lessons, calculators, and worksheets created to assist students and teachers of algebra.

Students
Begin by reading one of our lessons. Then find help with your homework using our step-by-step calculators, and test your knowledge using interactive worksheets. All content on algebrahelp.com is accessible free of charge.

Teachers
How do you plan to use Algebrahelp.com in the upcoming school year? We'd love to hear your stories with others on the website. Please let us know.

Six Million Equations Processed
The current release of the Equation Calculator has solved over six million equations! Thanks for helping to make this a big success!

More Worksheet Problems
At the request of our visitors, we have added more than 110 problems to our existing worksheets. We have also added a worksheet on Factoring Trinomials.

Survey
Help us build a better website! Please take a few minutes to complete the survey.
Teaching Math Vocabulary
Comprehending Math Vocabulary: The Barrier of Abstraction

“...when it comes to abstract mathematical concepts, words describe activities or relationships that often lack a visual counterpart. Yet studies show that children grasp the idea of quantity, as well as other relational concepts, from a very early age. As children develop their capacity for understanding, language, and its vocabulary, becomes a vital cognitive link between a child’s natural sense of number and order and conceptual learning.”

-Chard, D. (n.d.)

Math Vocabulary: Classroom (Tier I) Recommendations

- **Preteach math vocabulary.** Math vocabulary provides students with the language tools to grasp abstract mathematical concepts and to explain their own reasoning. Therefore, do not wait to teach that vocabulary only at ‘point of use’. Instead, preview relevant math vocabulary as a regular a part of the ‘background’ information that students receive in preparation to learn new math concepts or operations.

- **Model the relevant vocabulary when new concepts are taught.** Strengthen students’ grasp of new vocabulary by reviewing a number of math problems with the class, each time consistently and explicitly modeling the use of appropriate vocabulary to describe the concepts being taught. Then have students engage in cooperative learning or individual practice activities in which they too must successfully use the new vocabulary—while the teacher provides targeted support to students as needed.

- **Ensure that students learn standard, widely accepted labels for common math terms and operations and that they use them consistently to describe their math problem-solving efforts.**

Vocabulary: Why This Instructional Goal is Important

As vocabulary terms become more specialized in content area courses, students are less able to derive the meaning of unfamiliar words from context alone.

Students must instead learn vocabulary through more direct means, including having opportunities to explicitly memorize words and their definitions.

Students may require 12 to 17 meaningful exposures to a word to learn it.
Promoting Math Vocabulary: Other Guidelines

- Create a standard list of math vocabulary for each grade level (elementary) or course/subject area (for example, geometry).

- Periodically check students’ mastery of math vocabulary (e.g., through quizzes, math journals, guided discussion, etc.).

- Assist students in learning new math vocabulary by first assessing their previous knowledge of vocabulary terms (e.g., protractor, product) and then using that past knowledge to build an understanding of the term.

- For particular assignments, have students identify math vocabulary that they don’t understand. In a cooperative learning activity, have students discuss the terms. Then review any remaining vocabulary questions with the entire class.

- Encourage students to use a math dictionary in their vocabulary work.

- Make vocabulary a central part of instruction, curriculum, and assessment—rather than treating as an afterthought.


www.interventioncentral.org
Math Instruction: Unlock the Thoughts of Reluctant Students Through Class Journaling

Students can effectively clarify their knowledge of math concepts and problem-solving strategies through regular use of class ‘math journals’.

- At the start of the year, the teacher introduces the journaling weekly assignment in which students respond to teacher questions.
- At first, the teacher presents ‘safe’ questions that tap into the students’ opinions and attitudes about mathematics (e.g., ‘How important do you think it is nowadays for cashiers in fast-food restaurants to be able to calculate in their head the amount of change to give a customer?’). As students become comfortable with the journaling activity, the teacher starts to pose questions about the students’ own mathematical thinking relating to specific assignments. Students are encouraged to use numerals, mathematical symbols, and diagrams in their journal entries to enhance their explanations.
- The teacher provides brief written comments on individual student entries, as well as periodic oral feedback and encouragement to the entire class.
- Teachers will find that journal entries are a concrete method for monitoring student understanding of more abstract math concepts. To promote the quality of journal entries, the teacher might also assign them an effort grade that will be calculated into quarterly math report card grades.

Teaching Math Symbols
Learning Math Symbols: 3 Card Games

1. The interventionist writes math symbols that the student is to learn on index cards. The names of those math symbols are written on separate cards. The cards can then be used for students to play matching games or to attempt to draw cards to get a pair.

2. Create a card deck containing math symbols or their word equivalents. Students take turns drawing cards from the deck. If they can use the symbol/word on the selected card to formulate a correct ‘mathematical sentence’, the student wins the card. For example, if the student draws a card with the term ‘negative number’ and says that “A negative number is a real number that is less than 0”, the student wins the card.

3. Create a deck containing math symbols and a series of numbers appropriate to the grade level. Students take turns drawing cards. The goal is for the student to lay down a series of cards to form a math expression. If the student correctly solves the expression, he or she earns a point for every card laid down.

Use Visual Representations in Math Problem-Solving
Encourage Students to Use Visual Representations to Enhance Understanding of Math Reasoning

- Students should be taught to use standard visual representations in their math problem solving (e.g., numberlines, arrays, etc.)
- Visual representations should be explicitly linked with “the standard symbolic representations used in mathematics” p. 31
- Concrete manipulatives can be used, but only if visual representations are too abstract for student needs.

Examples of Math Visual Representations

Example 4. Representation of the counting on strategy using a number line

Example 5. Using visual representations for multidigit addition

A group of ten can be drawn with a long line to indicate that ten ones are joined to form one ten:

Simple drawings help make sense of two-digit addition with regrouping:

Response to Intervention

Schools Should Build Their Capacity to Use Visual Representations in Math

Caution: Many intervention materials offer only limited guidance and examples in use of visual representations to promote student learning in math.

Therefore, schools should increase their capacity to coach interventionists in the more extensive use of visual representations. For example, a school might match various types of visual representation formats to key objectives in the math curriculum.

Teach Students to Identify Underlying Structures of Math Problems
Teach Students to Identify ‘Underlying Structures’ of Word Problems

Students should be taught to classify specific problems into problem-types:

− Change Problems: Include increase or decrease of amounts. These problems include a time element

− Compare Problems: Involve comparisons of two different types of items in different sets. These problems lack a time element.

Teach Students to Identify ‘Underlying Structures’ of Word Problems

Change Problems: Include increase or decrease of amounts. These problems include a time element.

Example: Michael gave his friend Franklin 42 marbles to add to his collection. After receiving the new marbles, Franklin had 103 marbles in his collection.

How many marbles did Franklin have before Michael’s gift?

A 42 B ?

C 103

Teach Students to Identify ‘Underlying Structures’ of Word Problems

Compare Problems: Involve comparisons of two different types of items in different sets. These problems lack a time element.

Example: In the zoo, there are 12 antelopes and 17 alligators. How many more alligators than antelopes are there in the zoo?