Math Interventions. What are examples of math interventions that can be used in the classroom?
How Do We Reach Low-Performing Math Students?: Instructional Recommendations

Important elements of math instruction for low-performing students:

- “Providing teachers and students with data on student performance”
- “Using peers as tutors or instructional guides”
- “Providing clear, specific feedback to parents on their children’s mathematics success”
- “Using principles of explicit instruction in teaching math concepts and procedures.” p. 51

Math Interventions:

1. Peer Tutoring in Math Computation
3. Cognitive/Metacognitive Strategies for Word Problems
Peer Tutoring in Math
Computation with Constant
Time Delay
Peer Tutoring in Math Computation with Constant Time Delay

• **DESCRIPTION:** This intervention employs students as reciprocal peer tutors to target acquisition of basic math facts (math computation) using constant time delay (Meneses & Gresham, 2009; Telecsan, Slaton, & Stevens, 1999). Each tutoring ‘session’ is brief and includes its own progress-monitoring component—making this a convenient and time-efficient math intervention for busy classrooms.
Peer Tutoring in Math Computation with Constant Time Delay

MATERIALS:

Student Packet: A work folder is created for each tutor pair. The folder contains:

- 10 math fact cards with equations written on the front and correct answer appearing on the back. NOTE: The set of cards is replenished and updated regularly as tutoring pairs master their math facts.
- Progress-monitoring form for each student.
- Pencils.
Peer Tutoring in Math Computation with Constant Time Delay

**PREPARATION:** To prepare for the tutoring program, the teacher selects students to participate and trains them to serve as tutors.

**Select Student Participants.** Students being considered for the reciprocal peer tutor program should at minimum meet these criteria (Telecsan, Slaton, & Stevens, 1999, Meneses & Gresham, 2009):

- Is able and willing to follow directions;
- Shows generally appropriate classroom behavior;
- Can attend to a lesson or learning activity for at least 20 minutes.
Peer Tutoring in Math Computation with Constant Time Delay

Select Student Participants (Cont.). Students being considered for the reciprocal peer tutor program should at minimum meet these criteria (Telecsan, Slaton, & Stevens, 1999, Menesses & Gresham, 2009):

- Is able to name all numbers from 0 to 18 (if tutoring in addition or subtraction math facts) and name all numbers from 0 to 81 (if tutoring in multiplication or division math facts).

- Can correctly read aloud a sampling of 10 math-facts (equation plus answer) that will be used in the tutoring sessions. (NOTE: The student does not need to have memorized or otherwise mastered these math facts to participate—just be able to read them aloud from cards without errors).

- [To document a deficit in math computation] When given a two-minute math computation probe to complete independently, computes fewer than 20 correct digits (Grades 1-3) or fewer than 40 correct digits (Grades 4 and up) (Deno & Mirkin, 1977).
Peer Tutoring in Math Computation: Teacher Nomination Form

Reciprocal Peer Tutoring in Math Computation: Teacher Nomination Form

Teacher: ___________________________  Classroom: ___________________________  Date: __________

Directions: Select students in your class that you believe would benefit from participation in a peer tutoring program to boost math computation skills. Write the names of your student nominees in the space provided below. Remember, students who are considered for the peer tutoring program should—at minimum—meet these criteria:

- Show generally appropriate classroom behaviors and follow directions.
- Can pay attention to a lesson or learning activity for at least 20 minutes.
- Are able to wait appropriately to hear the correct answer from the tutor if the student does not know the answer.
- Can correctly read aloud a sampling of 10 math facts (equation plus answer) that will be used in the tutoring sessions. (Note: The student does not need to have memorized or otherwise mastered these math facts to participate—just be able to read them aloud from cards without errors.)

<table>
<thead>
<tr>
<th>Number</th>
<th>Student Name</th>
<th>NOTES</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
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<td>3.</td>
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</tbody>
</table>
Tutoring Activity. Each tutoring ‘session’ last for 3 minutes. The tutor:

- **Presents Cards.** The tutor presents each card to the tutee for 3 seconds.

- **Provides Tutor Feedback.** [When the tutee responds correctly] The tutor acknowledges the correct answer and presents the next card.

  [When the tutee does not respond within 3 seconds or responds incorrectly] The tutor states the correct answer and has the tutee repeat the correct answer. The tutor then presents the next card.

- **Provides Praise.** The tutor praises the tutee immediately following correct answers.

- **Shuffles Cards.** When the tutor and tutee have reviewed all of the math-fact carts, the tutor shuffles them before again presenting cards.
Peer Tutoring in Math Computation with Constant Time Delay

**Progress-Monitoring Activity.** The tutor concludes each 3-minute tutoring session by assessing the number of math facts mastered by the tutee. The tutor follows this sequence:

- *Presents Cards.* The tutor presents each card to the tutee for 3 seconds.
- *Remains Silent.* The tutor does not provide performance feedback or praise to the tutee, or otherwise talk during the assessment phase.
- *Sorts Cards.* Based on the tutee’s responses, the tutor sorts the math-fact cards into ‘correct’ and ‘incorrect’ piles.
- *Counts Cards and Records Totals.* The tutor counts the number of cards in the ‘correct’ and ‘incorrect’ piles and records the totals on the tutee’s progress-monitoring chart.
Tutoring Integrity Checks. As the student pairs complete the tutoring activities, the supervising adult monitors the integrity with which the intervention is carried out. At the conclusion of the tutoring session, the adult gives feedback to the student pairs, praising successful implementation and providing corrective feedback to students as needed. NOTE: Teachers can use the attached form Peer Tutoring in Math Computation with Constant Time Delay: Integrity Checklist to conduct integrity checks of the intervention and student progress-monitoring components of the math peer tutoring.
# Peer Tutoring in Math Computation: Intervention Integrity Sheet: (Part 1: Tutoring Activity)

## Peer Tutoring in Math Computation with Constant Time Delay: Integrity Checklist

### Tutoring Session: Intervention Phase

Directions: Observe the tutor and tutee for a full intervention session. Use this checklist to record whether each of the key steps of the intervention were correctly followed.

<table>
<thead>
<tr>
<th>Correctly Carried Out?</th>
<th>Step</th>
<th>Tutor Action</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>__Y__N</td>
<td>1.</td>
<td>Promptly Initiates Session. At the start of the timer, the tutor immediately presents the first math-fact card.</td>
<td></td>
</tr>
<tr>
<td>__Y__N</td>
<td>2.</td>
<td>Presents Cards. The tutor presents each card to the tutee for 3 seconds.</td>
<td></td>
</tr>
<tr>
<td>__Y__N</td>
<td>3.</td>
<td>Provides Tutor Feedback. [When the tutee responds correctly] The tutor acknowledges the correct answer and presents the next card. [When the tutee does not respond within 3 seconds or responds incorrectly] The tutor states the correct answer and has the tutee repeat the correct answer. The tutor then presents the next card.</td>
<td></td>
</tr>
<tr>
<td>__Y__N</td>
<td>4.</td>
<td>Provides Praise. The tutor praises the tutee immediately following correct answers.</td>
<td></td>
</tr>
<tr>
<td>__Y__N</td>
<td>5.</td>
<td>Shuffles Cards. When the tutor and tutee have reviewed all of the math-fact cards, the tutor shuffles them before again presenting cards.</td>
<td></td>
</tr>
<tr>
<td>__Y__N</td>
<td>6.</td>
<td>Continues to the Timer. The tutor continues to present math-fact cards for tutee response until the timer rings.</td>
<td></td>
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</tbody>
</table>
Peer Tutoring in Math Computation: Intervention Integrity Sheet (Part 2: Progress-Monitoring)

<table>
<thead>
<tr>
<th>Correctly Carried Out?</th>
<th>Step</th>
<th>Tutor Action</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Y</em> _N</td>
<td>1.</td>
<td>Presents Cards. The tutor presents each card to the tutee for 3 seconds.</td>
<td></td>
</tr>
<tr>
<td><em>Y</em> _N</td>
<td>2.</td>
<td>Remains Silent. The tutor does not provide performance feedback or praise to the tutee, or otherwise talk during the assessment phase.</td>
<td></td>
</tr>
<tr>
<td><em>Y</em> _N</td>
<td>3.</td>
<td>Sorts Cards. The tutor sorts cards into 'correct' and 'incorrect' piles based on the tutee's responses.</td>
<td></td>
</tr>
<tr>
<td><em>Y</em> _N</td>
<td>4.</td>
<td>Counts Cards and RECORDS TOTALS. The tutor counts the number of cards in the 'correct' and 'incorrect' piles and records the totals on the tutee's progress-monitoring chart.</td>
<td></td>
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</tbody>
</table>
Peer Tutoring in Math Computation: Score Sheet

<table>
<thead>
<tr>
<th>Date</th>
<th>Cards Correct</th>
<th>Cards Incorrect</th>
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</table>
Housing Bubble Graphic: New York Times

23 September 2007

Housing Price Index = 171 in 2005

As Prices Soared, Warnings of a Bust...

MAY 2003 The Economist magazine publishes a survey on global property prices, “Another Bubble Fit to Burst.”

MAY 2004 The economist and real estate skeptic Dean Baker sells his two-bedroom condo in the Adams Morgan neighborhood in Washington because he believes the gains in home prices are unsustainable.


MAY 2005 Alan Greenspan says: “Without creating the overall national issue a bubble, it’s pretty clear that it’s an unsustainable underlying pattern.”


FEB. 2006 Ben S. Bernanke, the Federal Reserve chairman, says policy makers “expect the housing market to cool but not to change very sharply.”

Sources: Standard & Poor’s/CASE-Shiller Home Price Index, Bureau of Labor Statistics
When encountering math graphics, students may:

- expect the answer to be easily accessible when in fact the graphic may expect the reader to interpret and draw conclusions
- be inattentive to details of the graphic
- treat irrelevant data as ‘relevant’
- not pay close attention to questions before turning to graphics to find the answer
- fail to use their prior knowledge both to extend the information on the graphic and to act as a possible ‘check’ on the information that it presents.

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics

Students can be more savvy interpreters of graphics in applied math problems by applying the Question-Answer Relationship (QAR) strategy. Four Kinds of QAR Questions:

- **RIGHT THERE** questions are fact-based and can be found in a single sentence, often accompanied by 'clue' words that also appear in the question.
- **THINK AND SEARCH** questions can be answered by information in the text but require the scanning of text and making connections between different pieces of factual information.
- **AUTHOR AND YOU** questions require that students take information or opinions that appear in the text and combine them with the reader's own experiences or opinions to formulate an answer.
- **ON MY OWN** questions are based on the students' own experiences and do not require knowledge of the text to answer.

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

1. DISTINGUISHING DIFFERENT KINDS OF GRAPHICS. Students are taught to differentiate between common types of graphics: e.g., table (grid with information contained in cells), chart (boxes with possible connecting lines or arrows), picture (figure with labels), line graph, bar graph.

   Students note significant differences between the various graphics, while the teacher records those observations on a wall chart. Next, students are given examples of graphics and asked to identify which general kind of graphic each is.

Finally, students are assigned to go on a ‘graphics hunt’, locating graphics in magazines and newspapers, labeling them, and bringing to class to review.

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

2. INTERPRETING INFORMATION IN GRAPHICS. Students are paired off, with stronger students matched with less strong ones. The teacher spends at least one session presenting students with examples from each of the graphics categories.

The presentation sequence is ordered so that students begin with examples of the most concrete graphics and move toward the more abstract: Pictures > tables > bar graphs > charts > line graphs.

At each session, student pairs examine graphics and discuss questions such as: “What information does this graphic present? What are strengths of this graphic for presenting data? What are possible weaknesses?”

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

3. LINKING THE USE OF QARS TO GRAPhICS. Students are given a series of data questions and correct answers, with each question accompanied by a graphic that contains information needed to formulate the answer.

Students are also each given index cards with titles and descriptions of each of the 4 QAR questions: RIGHT THERE, THINK AND SEARCH, AUTHOR AND YOU, ON MY OWN.

Working in small groups and then individually, students read the questions, study the matching graphics, and ‘verify’ the answers as correct. They then identify the type question being asked using their QAR index cards.

Using Question-Answer Relationships (QARs) to Interpret Information from Math Graphics: 4-Step Teaching Sequence

4. **USING QAR’S WITH GRAPHICS INDEPENDENTLY.** When students are ready to use the QAR strategy independently to read graphics, they are given a laminated card as a reference with 6 steps to follow:

   A. Read the question,
   B. Review the graphic,
   C. Reread the question,
   D. Choose a QAR,
   E. Answer the question, and
   F. Locate the answer derived from the graphic in the answer choices offered.

   Students are strongly encouraged NOT to read the answer choices offered until they have first derived their own answer, so that those choices don’t short-circuit their inquiry.

Elements of Metacognitive Processes

“Self-instruction helps students to identify and direct the problem-solving strategies prior to execution. Self-questioning promotes internal dialogue for systematically analyzing problem information and regulating execution of cognitive strategies. Self-monitoring promotes appropriate use of specific strategies and encourages students to monitor general performance. [Emphasis added].”

p. 231

Solving an advanced math problem independently requires the coordination of a number of complex skills. The following strategies combine both cognitive and metacognitive elements (Montague, 1992; Montague & Dietz, 2009). First, the student is taught a 7-step process for attacking a math word problem (cognitive strategy). Second, the instructor trains the student to use a three-part self-coaching routine for each of the seven problem-solving steps (metacognitive strategy).
Cognitive Portion of Combined Problem Solving Approach

In the cognitive part of this multi-strategy intervention, the student learns an explicit series of steps to analyze and solve a math problem. Those steps include:

1. **Reading the problem.** The student reads the problem carefully, noting and attempting to clear up any areas of uncertainty or confusion (e.g., unknown vocabulary terms).

2. **Paraphrasing the problem.** The student restates the problem in his or her own words.

3. **‘Drawing’ the problem.** The student creates a drawing of the problem, creating a visual representation of the word problem.

4. **Creating a plan to solve the problem.** The student decides on the best way to solve the problem and develops a plan to do so.

5. **Predicting/Estimating the answer.** The student estimates or predicts what the answer to the problem will be. The student may compute a quick approximation of the answer, using rounding or other shortcuts.

6. **Computing the answer.** The student follows the plan developed earlier to compute the answer to the problem.

7. **Checking the answer.** The student methodically checks the calculations for each step of the problem. The student also compares the actual answer to the estimated answer calculated in a previous step to ensure that there is general agreement between the two values.
Metacognitive Portion of Combined Problem Solving Approach

The metacognitive component of the intervention is a three-part routine that follows a sequence of ‘Say’, ‘Ask, ‘Check’. For each of the 7 problem-solving steps reviewed above:

- The student first self-instructs by stating, or ‘saying’, the purpose of the step (‘Say’).
- The student next self-questions by ‘asking’ what he or she intends to do to complete the step (‘Ask’).
- The student concludes the step by self-monitoring, or ‘checking’, the successful completion of the step (‘Check’).
# Combined Cognitive & Metacognitive Elements of Strategy

**Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)**

<table>
<thead>
<tr>
<th>Cognitive Strategy Step</th>
<th>Metacognitive ‘Say-Ask-Check’ Prompt Targets</th>
<th>Sample Metacognitive ‘Say-Ask-Check’ Prompts</th>
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<tbody>
<tr>
<td>1. <strong>Read the problem.</strong></td>
<td>‘Say’ (Self-Instruction) Target: <em>The student reads and studies the problem carefully before proceeding.</em>&lt;br&gt;‘Ask’ (Self-Question) Target: <em>Does the student fully understand the problem?</em>&lt;br&gt;‘Check’ (Self-Monitor) Target: <em>Proceed only if the problem is understood.</em></td>
<td><strong>Say:</strong> “I will read the problem. I will reread the problem if I don’t understand it.”&lt;br&gt;<strong>Ask:</strong> “Now that I have read the problem, do I fully understand it?”&lt;br&gt;<strong>Check:</strong> “I understand the problem and will move forward.”</td>
</tr>
</tbody>
</table>
## Combined Cognitive & Metacognitive Elements of Strategy

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</table>
| 2. **Paraphrase the problem.** | ‘Say’ (Self-Instruction) Target: *The student restates the problem in order to demonstrate understanding.*  
‘Ask’ (Self-Question) Target: *Is the student able to paraphrase the problem?*  
‘Check’ (Self-Monitor) Target: *Ensure that any highlighted key words are relevant to the question.* | **Say:** “I will highlight key words and phrases that relate to the problem question.”  
“I will restate the problem in my own words.”  
**Ask:** “Did I highlight the most important words or phrases in the problem?”  
**Check:** “I found the key words or phrases that will help to solve the problem.” |
# Combined Cognitive & Metacognitive Elements of Strategy

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</table>
| 3. ‘Draw’ the problem.  | ‘Say’ (Self-Instruction) Target: *The student creates a drawing of the problem to consolidate understanding.*  
‘Ask’ (Self-Question) Target: Is there a match between the drawing and the problem?  
‘Check’ (Self-Monitor) Target: *The drawing includes in visual form the key elements of the math problem.* | Say: “I will draw a diagram of the problem.”  
Ask: “Does my drawing represent the problem?”  
Check: “The drawing contains the essential parts of the problem.” |
### Combined Cognitive & Metacognitive Elements of Strategy

#### Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

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<tbody>
<tr>
<td>4. Create a plan to solve the problem.</td>
<td>‘Say’ (Self-Instruction) Target: The student generates a plan to solve the problem. ‘Ask’ (Self-Question) Target: What plan will help the student to solve this problem? ‘Check’ (Self-Monitor) Target: The plan is appropriate to solve the problem.</td>
<td>Say: “I will make a plan to solve the problem.” Ask: “What is the first step of this plan? What is the next step of the plan?” Check: “My plan has the right steps to solve the problem.”</td>
</tr>
</tbody>
</table>
## Combined Cognitive & Metacognitive Elements of Strategy

### Table 1: ‘Say-Ask-Check’ Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)

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</thead>
<tbody>
<tr>
<td>5. Predict/estimate the Answer.</td>
<td>‘Say’ (Self-Instruction) Target: The student uses estimation or other strategies to predict or estimate the answer. ‘Ask’ (Self-Question) Target: What estimating technique will the student use to predict the answer? ‘Check’ (Self-Monitor) Target: The predicted/estimated answer used all of the essential problem information.</td>
<td><strong>Say:</strong> “I will estimate what the answer will be.” <strong>Ask:</strong> “What numbers in the problem should be used in my estimation?” <strong>Check:</strong> “I did not skip any important information in my estimation.”</td>
</tr>
<tr>
<td>Cognitive Strategy Step</td>
<td>Metacognitive ‘Say-Aask-Check’ Prompt Targets</td>
<td>Sample Metacognitive ‘Say-Aask-Check’ Prompts</td>
</tr>
<tr>
<td>-------------------------</td>
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<td>--------------------------------------------</td>
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</tbody>
</table>
| 6. **Compute the answer.** | ‘Say’ (Self-Instruction) Target: The student follows the plan to compute the solution to the problem.  
‘Ask’ (Self-Question) Target: Does the answer agree with the estimate?  
‘Check’ (Self-Monitor) Target: The steps in the plan were followed and the operations completed in the correct order. | **Say**: “I will compute the answer to the problem.”  
**Ask**: “Does my answer sound right?” “Is my answer close to my estimate?”  
**Check**: “I carried out all of the operations in the correct order to solve this problem.” |
## Combined Cognitive & Metacognitive Elements of Strategy

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</table>
| 7. Check the answer.    | ‘Say’ (Self-Instruction) Target: *The student reviews the computation steps to verify the answer.*  
‘Ask’ (Self-Question) Target: *Did the student check all the steps in solving the problem and are all computations correct?*  
‘Check’ (Self-Monitor) Target: *The problem solution appears to have been done correctly.* | Say: “I will check the steps of my answer.”  
Ask: “Did I go through each step in my answer and check my work?”  
Check: “” |
Response to Intervention

Math Interventions:

1. Cover-Copy-Compare: Math Facts
2. Explicit Time Drill
3. Peer Tutoring in Math Computation
4. Customized Math Self-Correction Checklists
6. Cognitive/Metacognitive Strategies for Word Problems

Group Activity: Math Interventions

At your tables:

• Consider the math intervention ideas shared here.

• Discuss how you might use one or more of these strategies in your classroom or school.