How To: Assess Mastery of Math Facts With CBM: Computation Fluency

Computation Fluency measures a student's accuracy and speed in completing 'math facts' using the basic number operations of addition, subtraction, multiplication, and division. Computation fluency in the elementary grades is a strong predictor of later success in higher-level math coursework (Gersten, Jordan, & Flojo, 2005).

For students to attain 'computational fluency', however, they must be both accurate and speedy in solving basic math facts--ideally through automatic recall (VanDerHeyden & Burns, 2008). In an influential report, the National Mathematics Advisory Panel (2008) stressed the need for students to become proficient in math facts, calling on schools to make it a priority to "develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts." (p. xix).

The Common Core Standards also recognize the importance of computation fluency. For example, a 4th-grade math standard in Number and Operations in Base Ten (CCSM.4.NBT.4) states that the student will "fluently add and subtract multi-digit whole numbers using the standard algorithm" (National Governors Association Center for Best Practices et al., 2010; p. 29). However, the challenge for teachers is to define specifically what level of performance is required to identify a student as fluent in compuation.

CBM-Computation Fluency is a brief, timed assessment that can indicate to teachers whether a student is developing computation fluency and is thus on track to master grade-appropriate math facts (basic computation problems). This assessment can be administered to an individual student or to larger groups. The student is given a worksheet containing math facts and is given 2 minutes to answer as many problems as possible. The worksheet is then collected and scored, with the student receiving credit for each correct digit in his or her answers. Teachers can then compare any student's performance to research norms to determine whether that student is at risk because of delayed computational skills (Burns, VanDerHeyden, & Jiban, 2006).

Computation Fluency Measures: How to Access Resources. Teachers who would like to screen their students in grades 1 through 6 for possible delays in computation skills can obtain these free Computation Fluency assessment resources: (1) materials for assessment, (2) guidelines for administration and scoring, and (3) research-based norms.

- *Materials for assessment*. Schools can customize their own CBM Computation Fluency assessment materials at no cost, using the Math Worksheet Generator, a free online application: *http://www.interventioncentral.org/teacher-resources/math-work-sheet-generator*

  This program generates printable student and examiner assessment sheets for CBM Computation Fluency.

- *Guidelines for administration and scoring.* Instructions for preparing, administering, and scoring CBM-Computation Fluency assessments appear later in this document:

- *Research-based norms.* A table, *Curriculum-Based Measurement: Computation Fluency Norms* is included in this document. The table contains fluency benchmarks for grades 1-6, drawn from several research studies (e.g., Burns, VanDerHeyden, & Jiban, 2006).

References

Burns, M. K., VanDerHeyden, A. M., & Jiban, C. L. (2006). Assessing the instructional level for mathematics: A comparison of methods. *School Psychology Review, 35*, 401-418.

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Curriculum-Based Measurement-Computation Fluency: Guidelines for Use

CBM-Computation Fluency: Description

CBM-Computation Fluency measures a student's accuracy and speed in completing 'math facts' using the basic number operations of addition, subtraction, multiplication, and division. CBM-Computation Fluency probes are 2-minute assessments of basic math facts that are scored for number of 'correct digits'.

There are 2 types of CBM math probes, single-skill worksheets (those containing like problems) and multiple-skill worksheets (those containing a mix of problems requiring different math operations). Single-skill probes give instructors good information about students' mastery of particular problem-types, while multiple-skill probes allow the teacher to test children's math competencies on a range of computational objectives during a single CBM session.

Both types of math probes can be administered either individually or to groups of students. The examiner hands the worksheet(s) out to those students selected for assessment. Next, the examiner reads aloud the directions for the worksheet. Then the signal is given to start, and students proceed to complete as many items as possible within 2 minutes. The examiner collects the worksheets at the end of the assessment for scoring.

CBM-Computation Fluency: Materials

The following materials are needed to administer CBM-Computation Fluency:

- Student and examiner copies of CBM Computation Fluency Probes
- Stopwatch
- Pencils for students

CBM-Computation Fluency: Preparation

After computational objectives have been selected, the instructor is ready to prepare math probes. The teacher may want to create single-skills probes, multiple-skill probes, or both types of CBM math worksheets. The teacher will probably want to consult the Common Core State Standards for Mathematics or district math curriculum when selecting the kinds of problems to include in the single- or multiple-skill probe.

Creating the single-skill math probe. As the first step in putting together a single-skill math probe, the teacher will select one computational objective as a guide. The worksheet, then, will consist of problems randomly constructed that conform to the computational objective chosen.

Figure 1: A Sampling of Math Computational Goals for Addition, Subtraction, Multiplication, and Division (from Wright, 2002).

<u>Addition</u>
Two 1-digit numbers: sums to 10
Two 3-digit numbers: no regrouping
1- to 2-digit number plus 1- to 2-digit number: regrouping

<u>Subtraction</u>
Two 1-digit numbers: 0 to 9
2-digit number from a 2-digit number: no regrouping
2-digit number from a 2-digit number: regrouping

<u>Multiplication</u>
Multiplication facts: 0 to 9
2-digit number times 1-digit number: no regrouping
3-digit number times 1-digit number: regrouping

<u>Division</u>
Division facts: 0 to 9
2-digit number divided by 1-digit number: no remainder
2-digit number divided by 1-digit number: remainder

Wright, J. (2002) *Curriculum-Based Assessment Math Computation Probe Generator: Multiple-Skill Worksheets in Mixed Skills*. Retrieved from *http://www.interventioncentral.org/teacher-resources/math-work-sheet-generator*

For example, the instructor may select any of the computational objectives in Figure 1 as the basis for a math probe. The teacher would then construct a series of problems that match the computational goal, as in Figure 2. In general, single-skill math probes should contain between 80 and 200 problems, and worksheets should have items on both the front and back of the page. Adequate space should also be left for the student to show his or her work, especially with more complex problems such as long division.

Figure 2: Example of a single-skill math probe: Three to five 3- and 4-digit numbers: no regrouping

```
     105   |      2031   |      111   |       634   |
 +   600   |  +    531   |  +   717   |  + 8240     |
 +   293   |  +   2322   |  +   260   |  +   203    |
```

Creating the Multiple-skill Math Probe. To assemble a multiple-skill math probe, the instructor will first select the range of math operations and of problem-types that will make up the probe. Once the computational objectives have been

Figure 3: Example of a multiple-skill math probe:

- *Division: 3-digit number divided by 1-digit number: no remainder*
- *Subtraction: 2-digit number from a 2-digit number: regrouping*
- *Multiplication” 3-digit number times 1-digit number: no regrouping*
- *Division: Two 3-digit numbers: no regrouping*

```
          |    20   |   113   |      106   |
  9/431   |   -18   |   x  2  |  +   172   |
          |         |         |  +   200   |
          |         |         |  +   600   |
```

chosen, the teacher can make up a worksheet of mixed math facts conforming to those objectives. Using our earlier example, the teacher who wishes to estimate the proficiency of his 4th-grade math group may decide to create a multiple-skills CBM probe. He could choose to sample only those problem-types that his students have either mastered or are presently being taught. Figure 3 shows four computation skills with matching sample problems that might appear on a worksheet of mixed math facts.

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CBM-Computation Fluency: Directions for Administration

1. The examiner distributes copies of math probes to all the students in the group, face down. (Note: These probes may also be administered individually). The examiner says to the students: "The sheets on your desk are math facts."
2. If the students are to complete a single-skill probe, the examiner says: "All the problems are [addition or subtraction or multiplication or division] facts."

If the students are to complete a multiple-skill probe, the examiner then says: "There are several types of problems on the sheet. Some are addition, some are subtraction, some are multiplication, and some are division [as appropriate]. Look at each problem carefully before you answer it."

3. The examiner then says: "When I say 'begin', turn the worksheet over and begin answering the problems. Start on the first problem on the left on the top row [point]. Work across and then go to the next row. If you can't answer a problem, make an 'X' on it and go to the next one. If you finish one side, go to the back. Are there any questions? ".
4. The examiner says 'Start' and starts the stopwatch. While the students are completing worksheets, the examiner and any other adults assisting in the assessment circulate around the room to ensure that students are working on the correct sheet and that they are completing problems in the correct order (rather than picking out only the easy items)..
5. After 2 minutes have passed, the examiner says, "Stop" and collects the CBM computation probes for scoring.
6. *Initial Assessment:* If the examiner is assessing the student for the first time, the examiner administers a total of 3 computation probes during the session using the above procedures and takes the median (middle) score as the best estimate of the student's computation fluency.
   Progress-Monitoring: If the examiner is monitoring student growth in computation (and has previously collected CBM-Computation Fluency data), only one computation probe is given in the session.

CBM-Computation Fluency: Directions for Practice

If the student is not yet familiar with CBM-Computation Fluency probes, the teacher can administer one or more practice computation probes (using the administration guidelines above) and provide coaching and feedback as needed until assured that the student fully understands the assessment.

CBM-Computation Fluency: Scoring Guidelines

Traditional approaches to computational assessment usually give credit for the total number of correct answers appearing on a worksheet. If the answer to a problem is found to contain one or more incorrect digits, that problem is marked wrong and receives no credit. In contrast to this all-or-nothing marking system, CBM assigns credit to each individual correct digit appearing in the solution to a math fact.

On the face of it, a math scoring system that awards points according to the number of correct digits may appear unusual, but this alternative approach is grounded in good academic-assessment research and practice. By separately scoring each digit in the answer of a computation problem, the instructor is better able to recognize and to give credit for a student's partial math competencies. Scoring computation problems by the digit rather than as a single answer also allows for a more minute analysis of a child's number skills.

Imagine, for instance, that a student was given a CBM math probe consisting of addition problems, sums less than or equal to 19 (incorrect digits appear in boldface and italics):

Figure 4: Example of completed problems from a single-skill math probe

105	2031	111	634
+ 600	+ 531	+ 717	+ 8240
+ 293	+ 2322	+ 260	+ 203
9***8***8	4884	108***7***	9***0***77

If the answers in Figure 4 were scored as either correct or wrong, the child would receive a score of 1 correct answer out of 4 possible answers (25 percent). However, when each individual digit is scored, it becomes clear that the student actually correctly computed 12 of 15 possible digits (80 percent). Thus, the CBM procedure of assigning credit to each correct digit demonstrates itself to be quite sensitive to a student's emerging, partial competencies in math computation.

The following scoring rules will aid the instructor in marking single- and multiple-skill math probes:

- Individual correct digits are counted as correct.
  Reversed or rotated digits are not counted as errors unless their change in position makes them appear to be another digit (e.g., 9 and 6).

- Incorrect digits are counted as errors.
  Digits that appear in the wrong place value, even if otherwise correct, are scored as errors.

Example

```
  97
  x9
----
8730  <--
```

"873" is the correct answer to this problem, but no credit can be given since the addition of the 0 pushes the other digits out of their proper place-value positions.

- The student is given credit for "place-holder" numerals that are included simply to correctly align the problem. As long as the student includes the correct space, credit is given whether or not a "0" has actually been inserted.

Example

```
  55
x 82
----
 110
4400  <--
----
4510
```

Since the student correctly placed 0 in the "place-holder" position, it is given credit as a correct digit. Credit would also have been given if the space were reserved but no 0 had been inserted.

- In more complex problems such as advanced multiplication, the student is given credit for all correct numbers that appear below the line.

Example

```
  33
x 28
----
 264
 660
----
 924  <--
```

Credit is given for all work below the line. In this example, the student earns credit for 9 correct digits.

- Credit is not given for any numbers appearing above the line (e.g., numbers marked at the top of number columns to signify regrouping).

Example

```
 1   <--
  46
+ 39
----
  85
```

Credit is given for the 2 digits below the line. However, the carried "1" above the line does not receive credit.

Curriculum-Based Measurement: Computation Fluency Norms

(Burns, VanDerHeyden, & Jiban, 2006; Deno & Mirkin, 1977; Fuchs & Fuchs, 1993; Fuchs & Fuchs, n.d.)*

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NOTE: The norms for grades 2-5 presented below are for 1 minute, while the norms for grades 1 and 6 are for 2 minutes. To use any of the 1-minute norms, (1) administer and score a standard 2-minute Computation Fluency probe; (2) divide that student score by 2; and then (3) compare that converted student score to the appropriate 1-minute norm within grades 2-5 (Burns, VanDerHeyden, & Jiban, 2006).

Grade	End of Year Benchmark: Correct Digits per 2 Mins (Fuchs & Fuchs, n.d.)	Weekly Growth: 'Realistic' (Fuchs & Fuchs, 1993)	Weekly Growth: 'Ambitious' (Fuchs & Fuchs, 1993)
1	20	0.3	0.5

Grade	Performance Level	Correct Digits per 1 Min (Burns, VanDerHeyden, & Jiban, 2006)	Weekly Growth: 'Realistic' (Fuchs & Fuchs, 1993)	Weekly Growth: 'Ambitious' (Fuchs & Fuchs, 1993)
2	Mastery	More than 31	0.3	0.5
	Instructional	14-31		
	Frustration	Less than 14		
3	Mastery	More than 31	0.3	0.5
	Instructional	14-31		
	Frustration	Less than 14		
4	Mastery	More than 49	0.75	1.2
	Instructional	24-49		
	Frustration	Less than 24		
5	Mastery	More than 49	0.75	1.2
	Instructional	24-49		
	Frustration	Less than 24		

Grade	Performance Level	Correct Digits per 2 Mins (Deno & Mirkin, 1977)	Weekly Growth: 'Realistic' (Fuchs & Fuchs, 1993)	Weekly Growth: 'Ambitious' (Fuchs & Fuchs, 1993)
6	Mastery	More than 79	0.45	1.0
	Instructional	40-79		
	Frustration	Less than 40		

References:

- Burns, M. K., VanDerHeyden, A. M., & Jiban, C. L. (2006). Assessing the instructional level for mathematics: A comparison of methods. *School Psychology Review, 35*, 401-418.
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*Reported Characteristics of Student Sample(s) Used to Compile These Norms:

- Burns, VanDerHeyden, & Jiban, 2006: *Number of Students Assessed:* 434 students across grades 2-5/*Geographical Location:* Southwest: Sample drawn from 1 elementary school/ *Socioeconomic Status:* 15% rate of Free & Reduced Lunch/ *Ethnicity of Sample:* 74% Caucasian-non-Hispanic; 17% Hispanic or Latino; 6% African-American; 3% Asian-American; 1% Native American/*Limited English Proficiency in Sample:* 2% of students.
- Deno & Mirkin, 1977: *Number of Students Assessed:* Not reported/*Geographical Location:* Sample drawn from 1 elementary school; location not reported/ *Socioeconomic Status:* Not reported/ *Ethnicity of Sample:* Not reported/*Limited English Proficiency in Sample:* Not reported.
- Fuchs & Fuchs, n.d.: *Number of Students Assessed:* Not reported/*Geographical Location:* Not reported/ *Socioeconomic Status:* Not reported/ *Ethnicity of Sample:* Not reported/*Limited English Proficiency in Sample:* Not reported.
- Fuchs & Fuchs, 1993: *Number of Students Assessed:* Year 1: 177 students in grades 1-6; Year 2:1208 students across grades 1-6/*Geographical Location:* Upper Midwest: Sample drawn from 5 elementary schools/ *Socioeconomic Status:* 33%-55% rate of Free & Reduced Lunch across participating schools/ *Ethnicity of Sample:* Not reported/*Limited English Proficiency in Sample:* Not reported.

Where to Find Materials: Schools can create their own CBM Computation Fluency assessment materials at no cost, using the Math Worksheet Generator, a free online application:
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This program generates printable student and examiner assessment sheets for CBM Computation Fluency.

Limitations of These Research Norms: Norms generated from small-scale research studies--like those used here--provide estimates of student academic performance based on a sampling from only one or two points in time, rather than a more comprehensive sampling across separate fall, winter, and spring screenings. These norms also have been compiled from a relatively small student sample that is not fully representative of a diverse 'national' population. Nonetheless, norms such as these are often the best information that is publically available for basic academic skills and therefore do have a definite place in classroom instruction decision-making.

These norms can be useful in general education for setting student performance outcome goals for core instruction and/or any level of academic intervention. Similarly, these norms can be used to set performance goals for students with special needs. In both cases, however, single-sample norms would be used only if more comprehensive fall/winter/spring academic performance norms are not available.